**Appendix 1: Two-Link Planar Control simulation results**

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# Simulation parameters

## Robot parameters

Table 1: Robot parameter

|  |  |
| --- | --- |
| Parameter | Value |
| g | 9.8066 (in Y-direction) |
| Link density | 3700 kg/m3 |
| Link 1 dimensions (x\*y\*z) | 0.3m\*0.05m\*0.05m |
| Link 2 dimensions (x\*y\*z) | 0.15m\*0.05m\*0.05m |
| m1 | 2.775 kg |
| m2 | 1.3875 kg |
| b1 | 0.01 |
| b2 | 0.01 |
| Initial values ( | All zeros |

## Realistic simulation parameters

* Gaussian measurement noise is added to joint position with standard deviation equal to position precision
* Only joint noise is added since it will propagate through the integrals to velocity and acceleration and only position sensor is used.
* A first-order low-pass filter is added to smooth joint position measurements (introduces some lag)

Table 2: Simulation parameters (realistic)

|  |  |
| --- | --- |
| Parameter | Value |
| Sampling time | 1ms |
| Maximum velocity |  |
| Maximum joint 1 torque | 20 Nm |
| Maximum joint 2 torque | 10 Nm |
| Joint position precision |  |
| Robot parameters deviation (mass, length, viscosity)  (Used in the dynamics terms of some controllers) |  |

# Simulation results (ideal)

## PD Joint control

Table 3: PD gains

|  |  |  |  |
| --- | --- | --- | --- |
| Parameter | Value | Parameter | Value |
|  | 30 |  | 7 |
|  | 2.5 |  | 0.5 |

### Step response

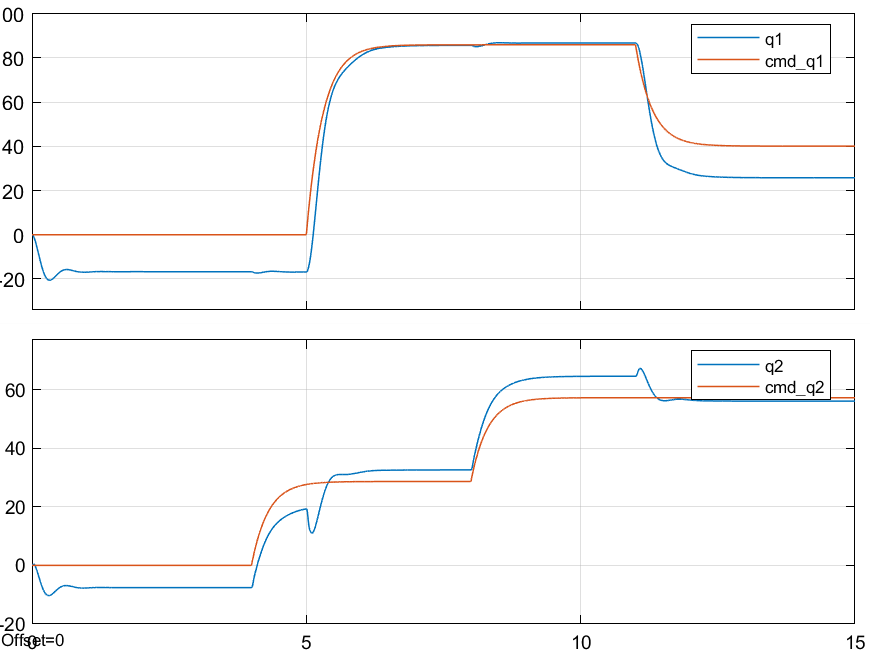


Figure 1: PD control ideal system step response - joint position

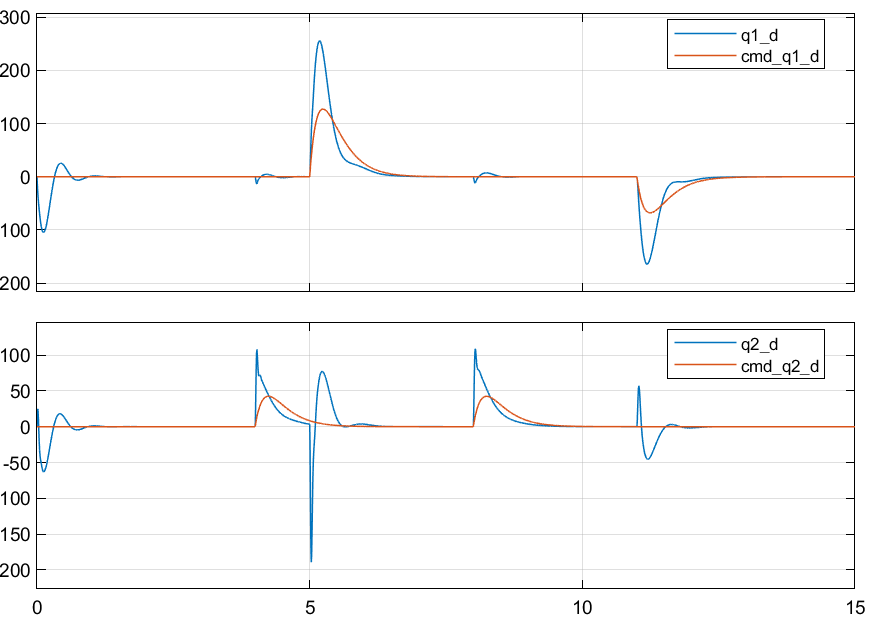


Figure 2: PD control ideal system step response - joint velocity

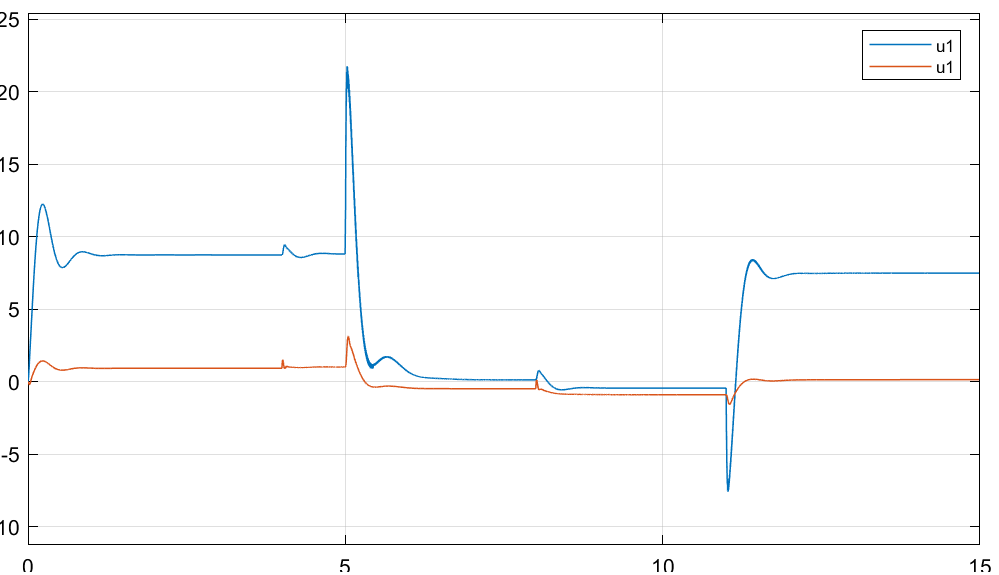


Figure 3: PD control ideal system step response - joint input torque

### Sine wave response

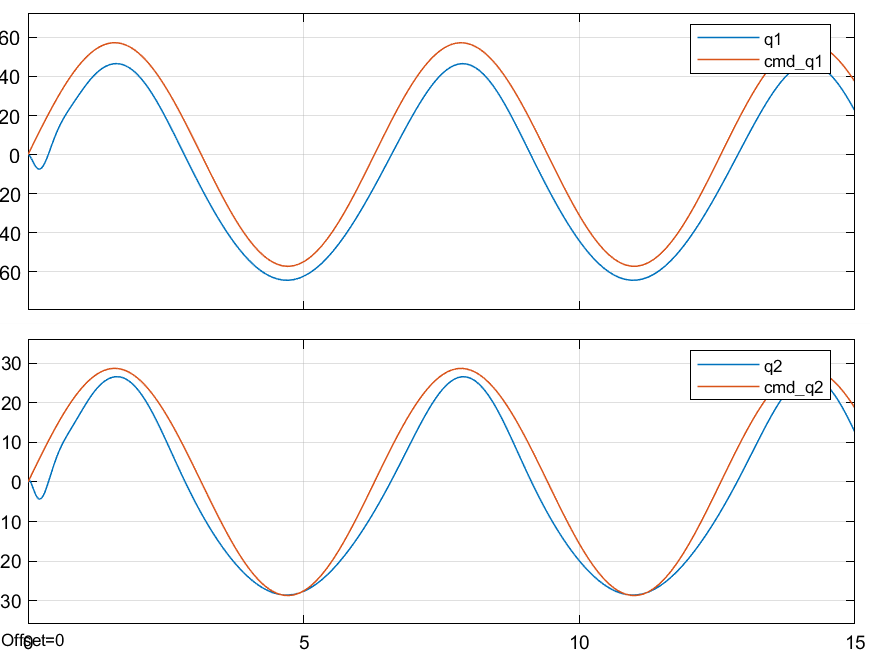


Figure 4: PD control ideal system sine wave response - joint position

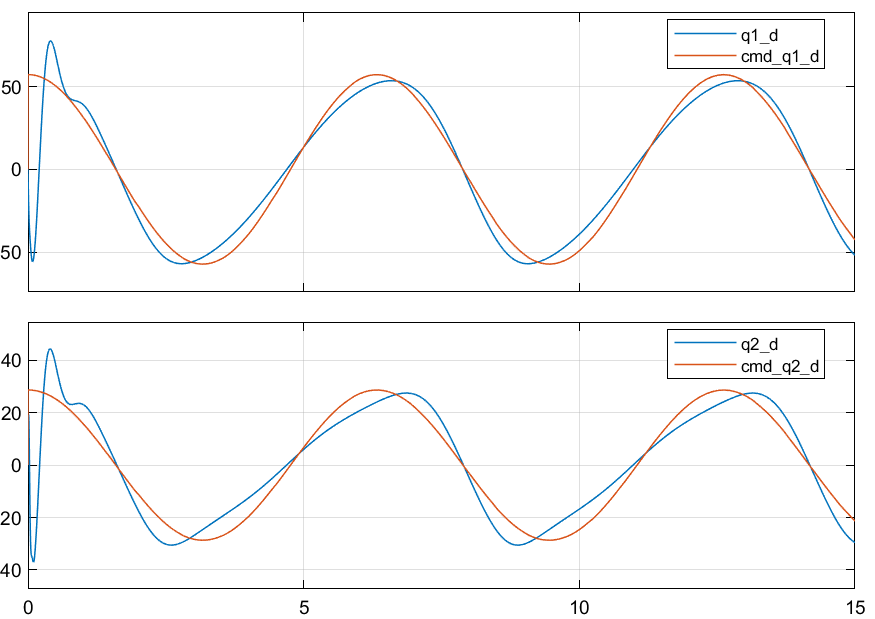


Figure 5: PD control ideal system sine wave response - joint velocity

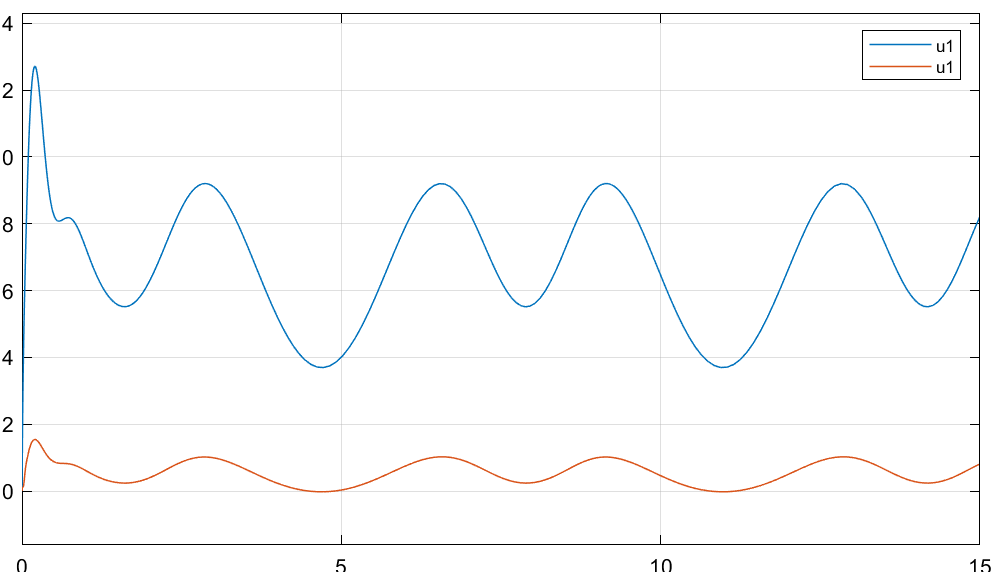


Figure 6: PD control ideal system sine wave response - joint input torque

### Observations

* Nonzero steady-state and tracking errors as expected

## PID Joint control

Table 4: PID gains

|  |  |  |  |
| --- | --- | --- | --- |
| Parameter | Value | Parameter | Value |
|  | 30 |  | 7 |
|  | 2.5 |  | 0.5 |
|  | 20 |  | 15 |

### Step response

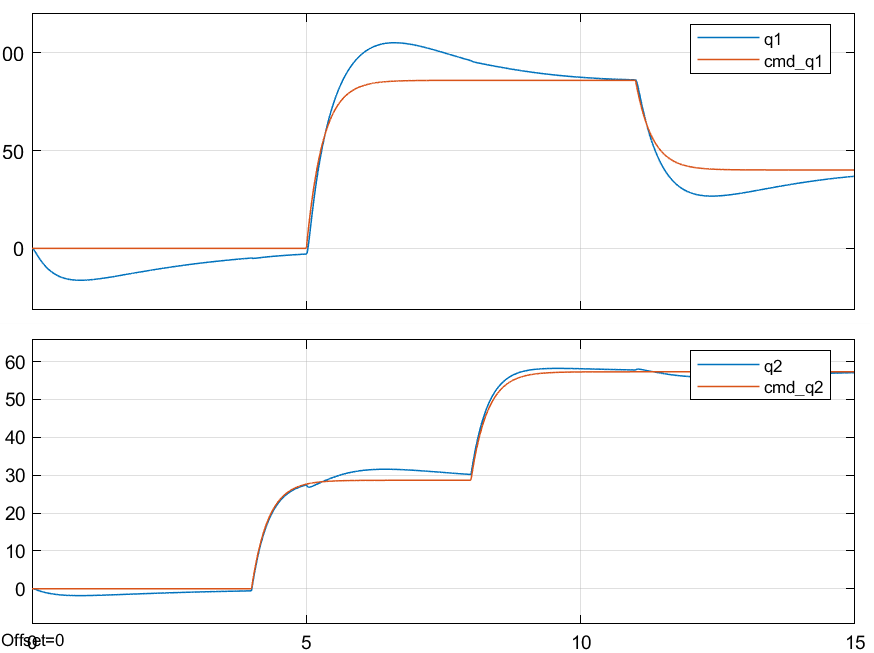


Figure 7: PID control ideal system step response - joint position

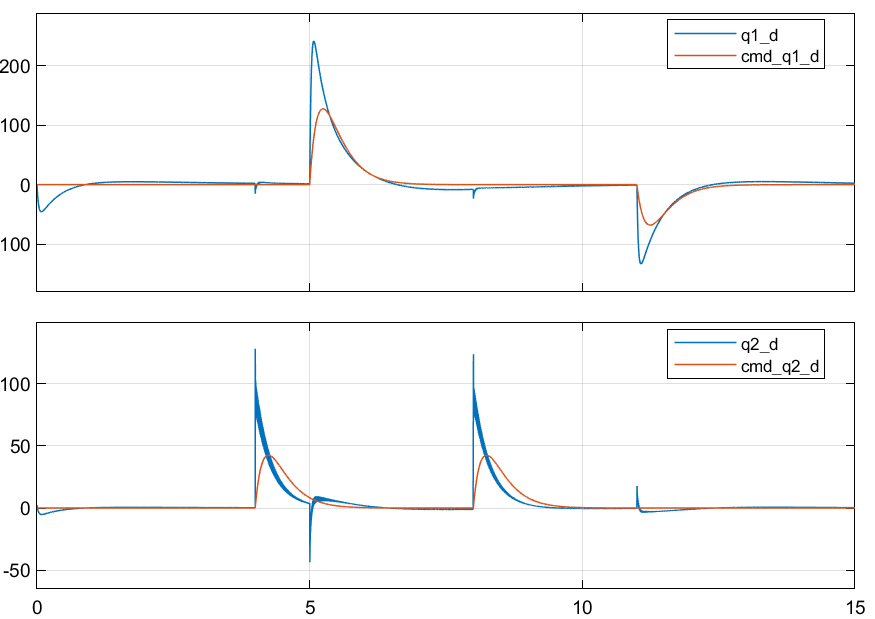


Figure 8: PID control ideal system step response - joint velocity

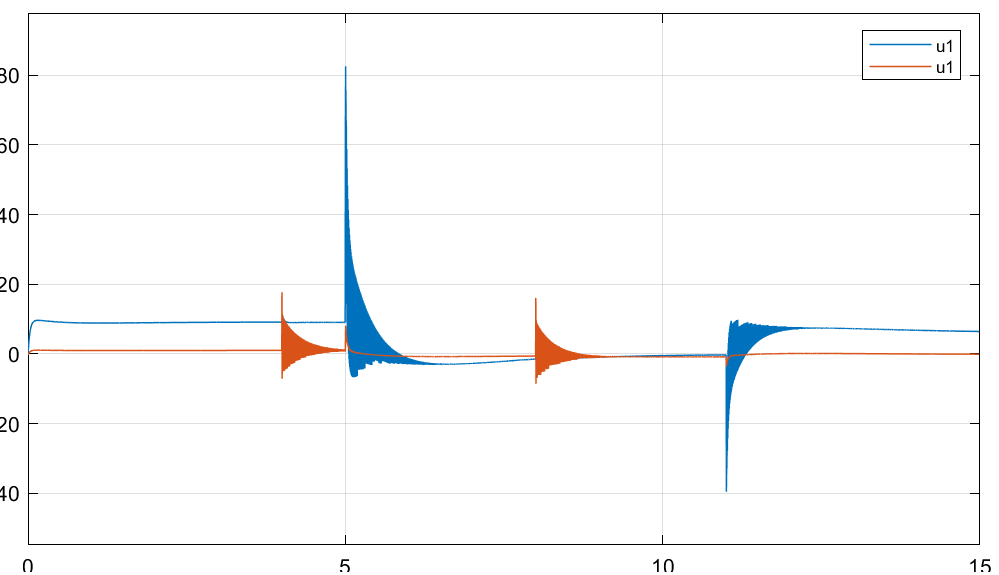


Figure 9: PID control ideal system step response - joint input torque

### Sine wave response

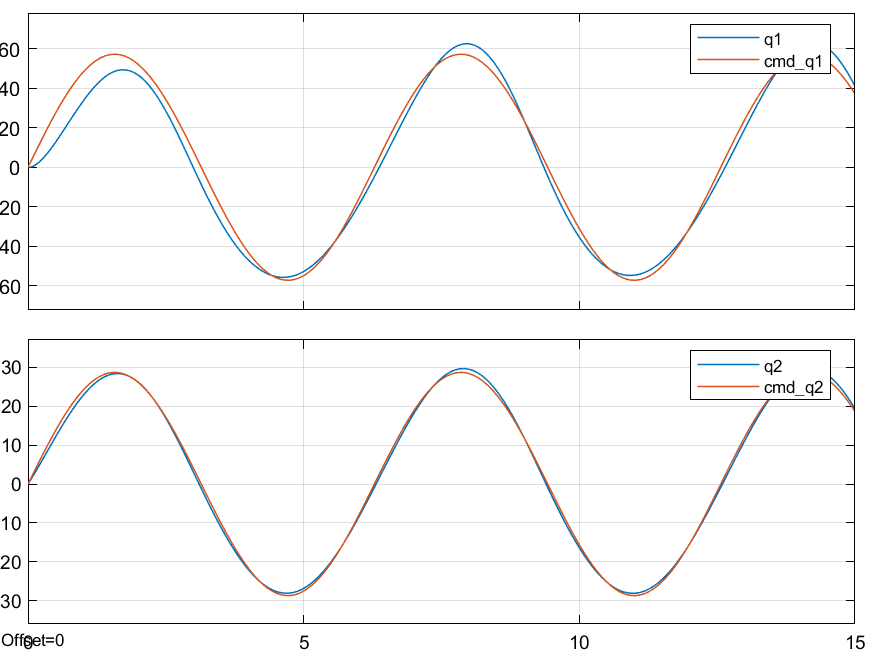


Figure 10: PID control ideal system sine wave response - joint position

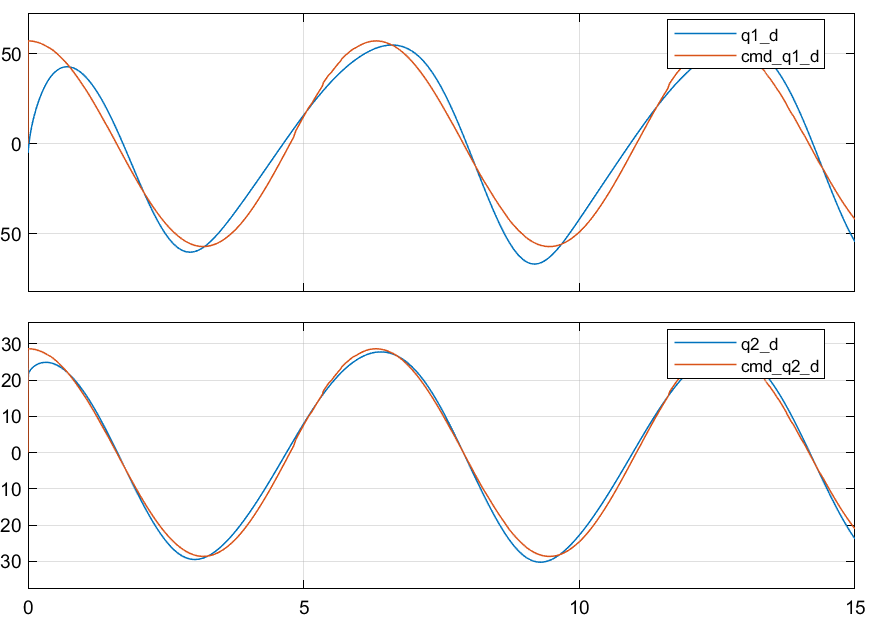


Figure 11: PID control ideal system sine wave response - joint velocity

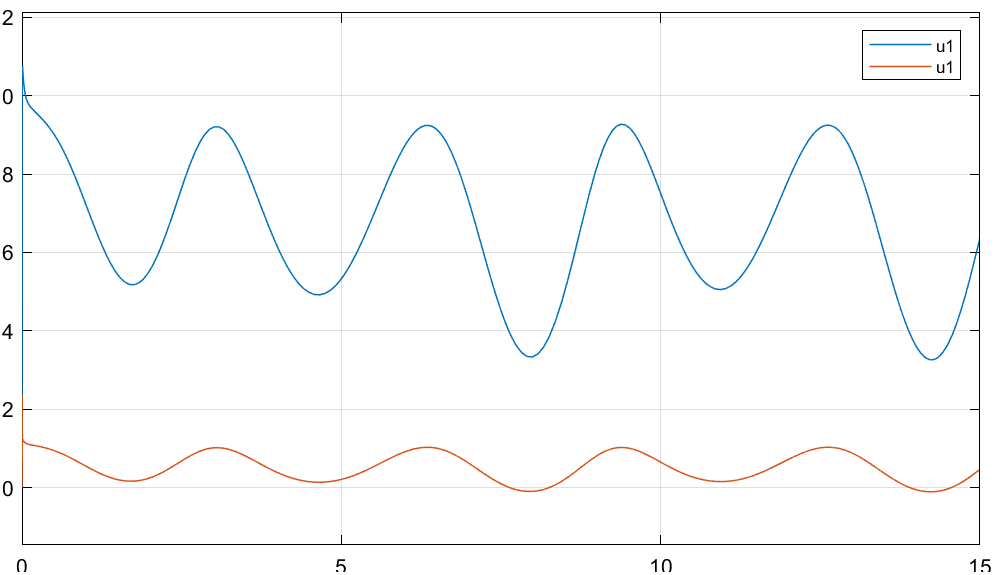


Figure 12: PID control ideal system sine wave response - joint input torque

### Observations

* Steady-state goes to zero when given enough time
* Tracking error is still not zero due to response lag
* High-magnitude and high frequency commanded torques at the step edge

## Pole Placement Control

Table 5: Pole values and feedback gain- ideal

|  |  |
| --- | --- |
| Parameter | Value |
| poles | [-90, -80, -70, -10.8] |
|  |  |

### Step response

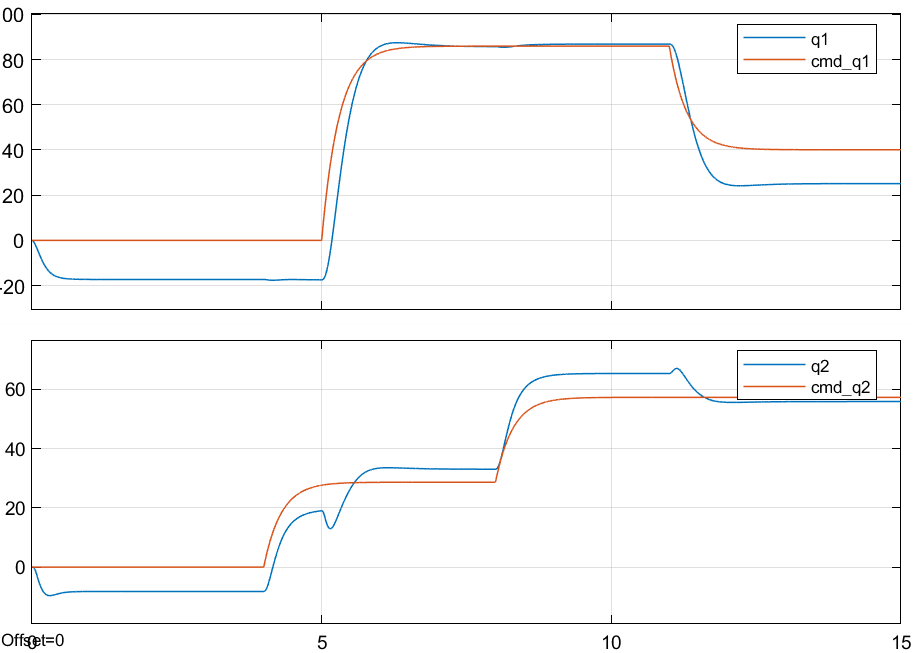


Figure 13: Pole placement control ideal system step response - joint position

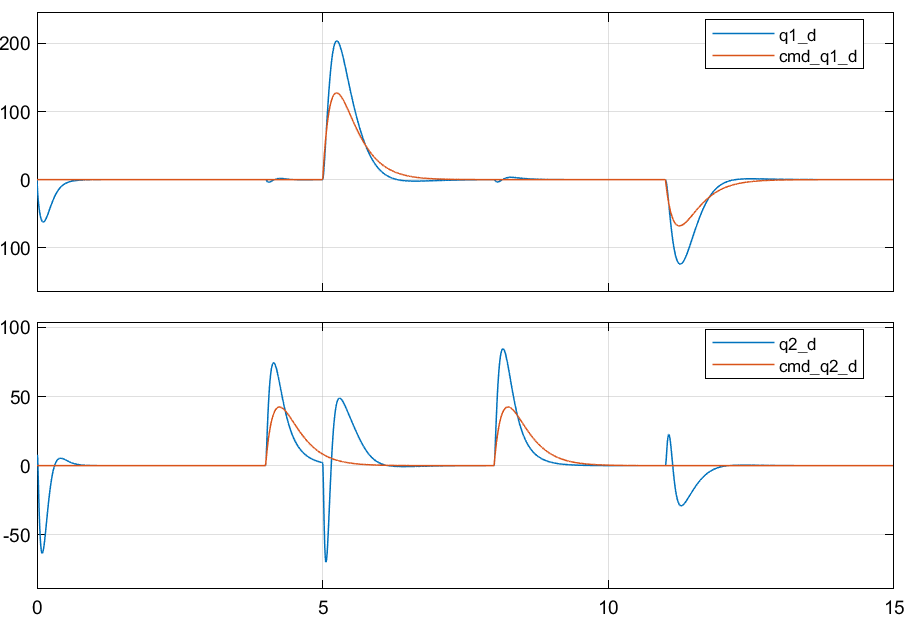
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Figure 14: Pole placement control ideal system step response - joint velocity

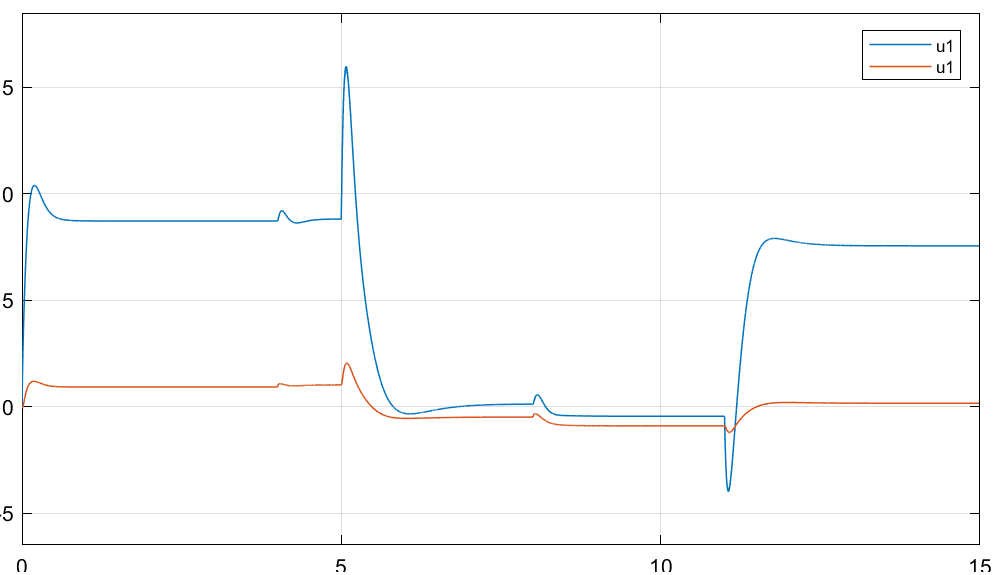


Figure 15: Pole placement control ideal system step response - joint input torque

### Sine wave response

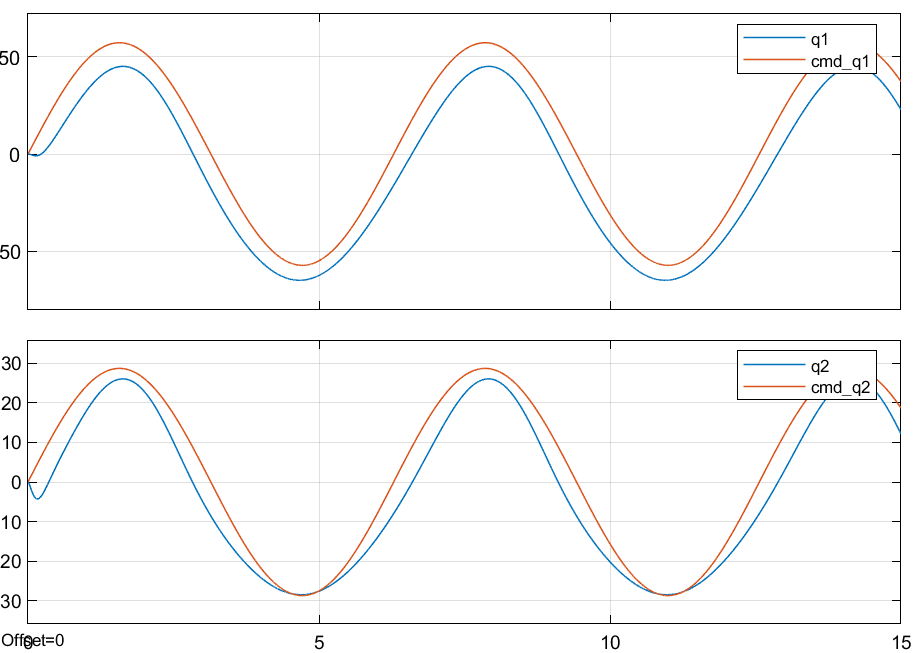


Figure 16: Pole placement control ideal system sine wave response - joint position

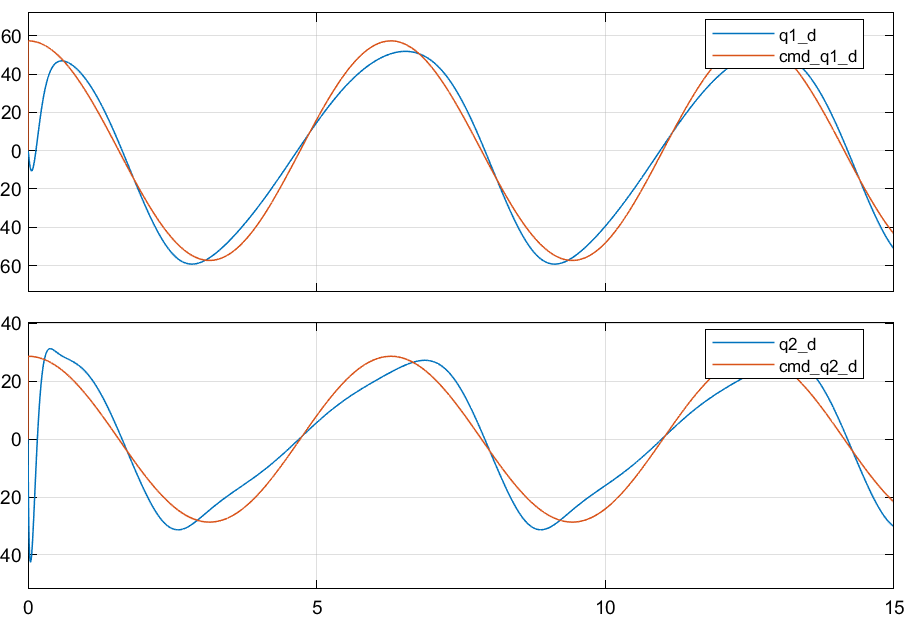


Figure 17: Pole placement control ideal system sine wave response - joint velocity

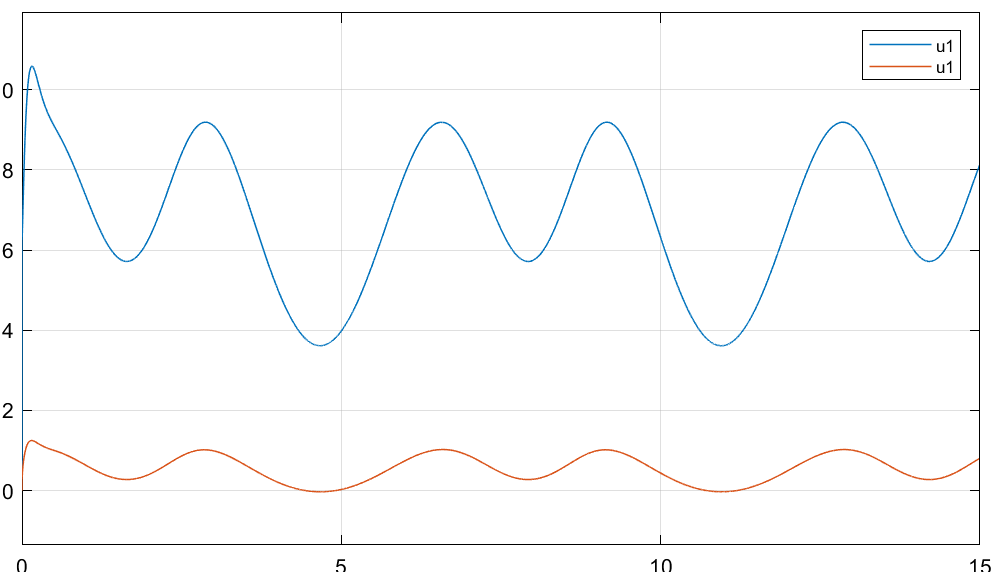


Figure 18: Pole placement control ideal system sine wave response - joint input torque

### Observations

* Nonzero Steady-state and tracking error

## Pole Placement Control with feedforward disturbance terms

### Step response

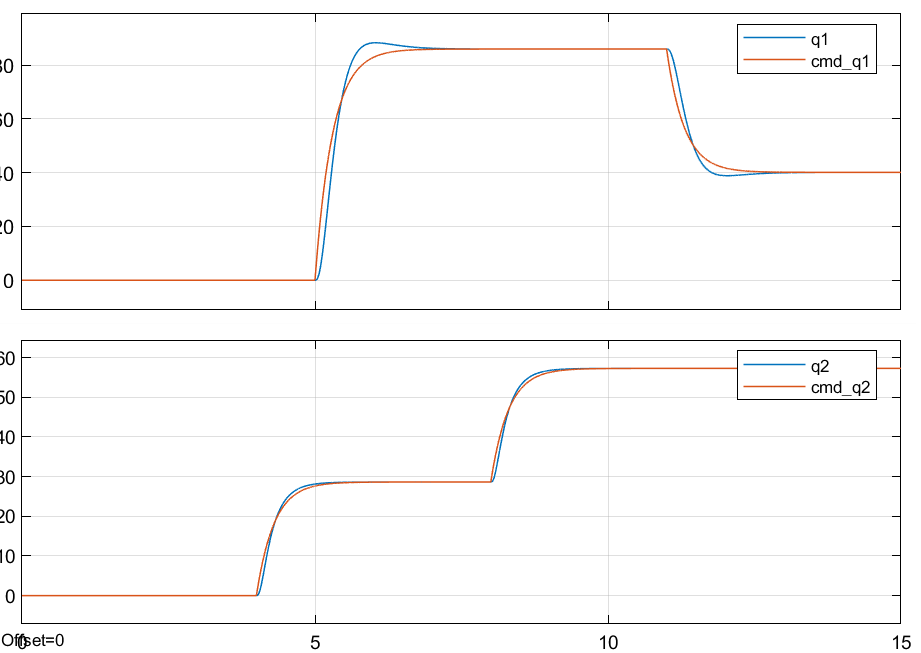


Figure 19: Pole placement+D control ideal system step response - joint position

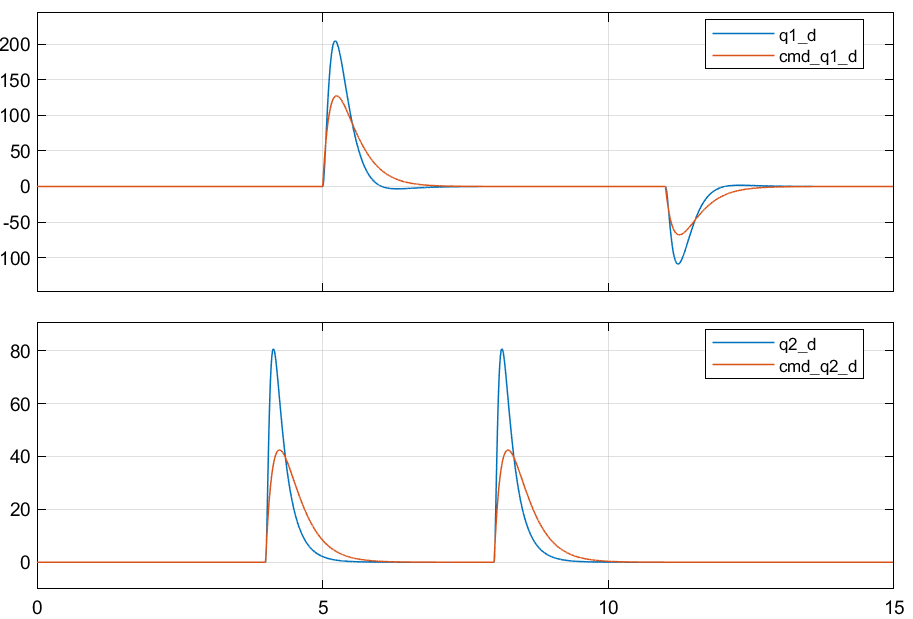
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Figure 20: Pole placement+D control ideal system step response - joint velocity

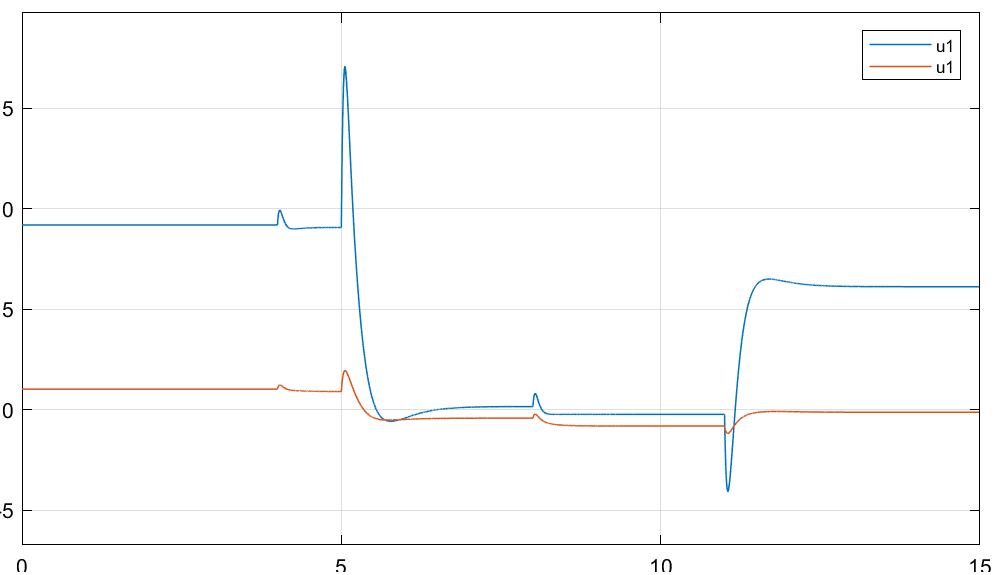


Figure 21: Pole placement+D control ideal system step response - joint input torque

### Sine wave response

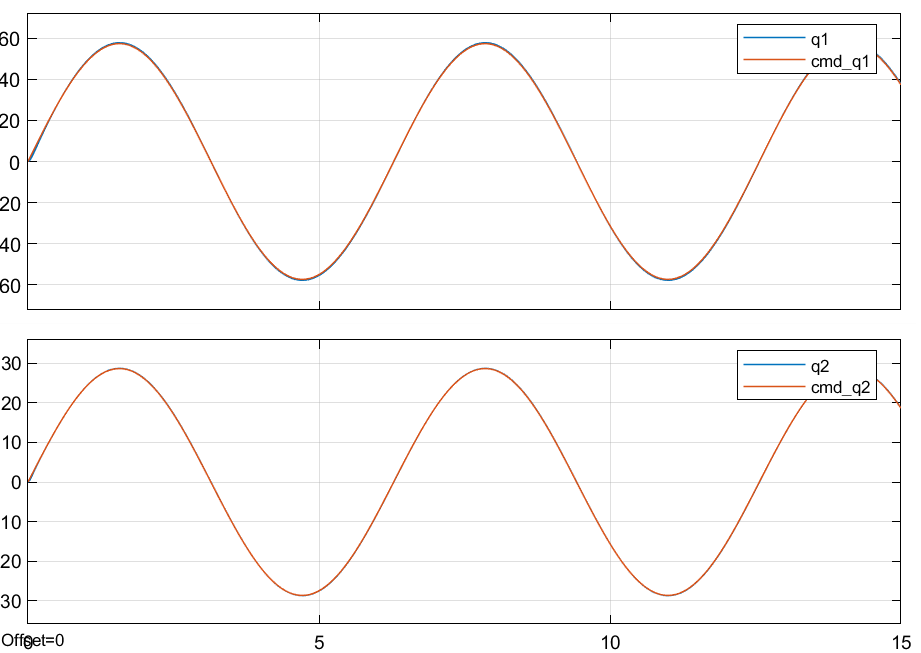


Figure 22: Pole placement+D control ideal system sine wave response - joint position

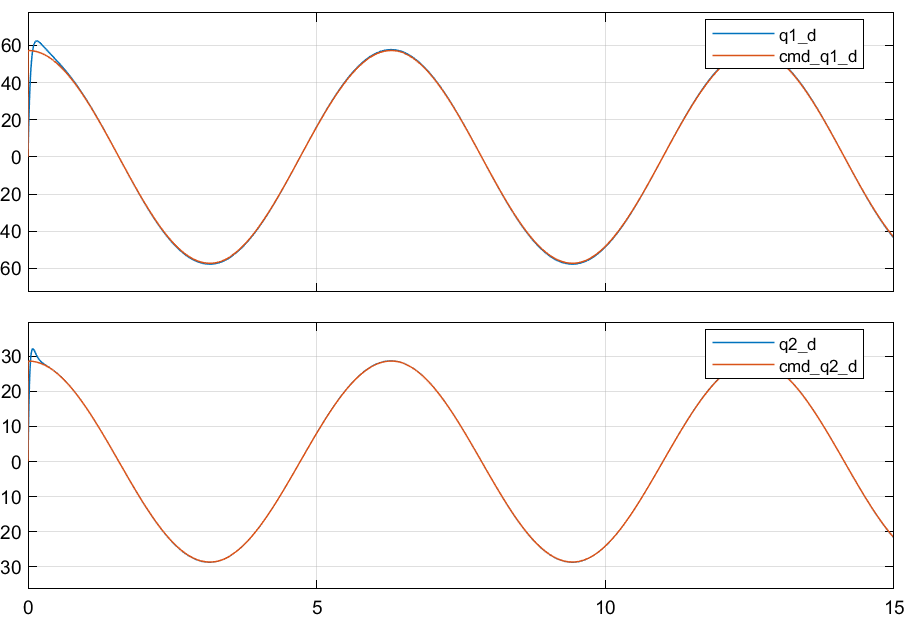


Figure 23: Pole placement+D control ideal system sine wave response - joint velocity

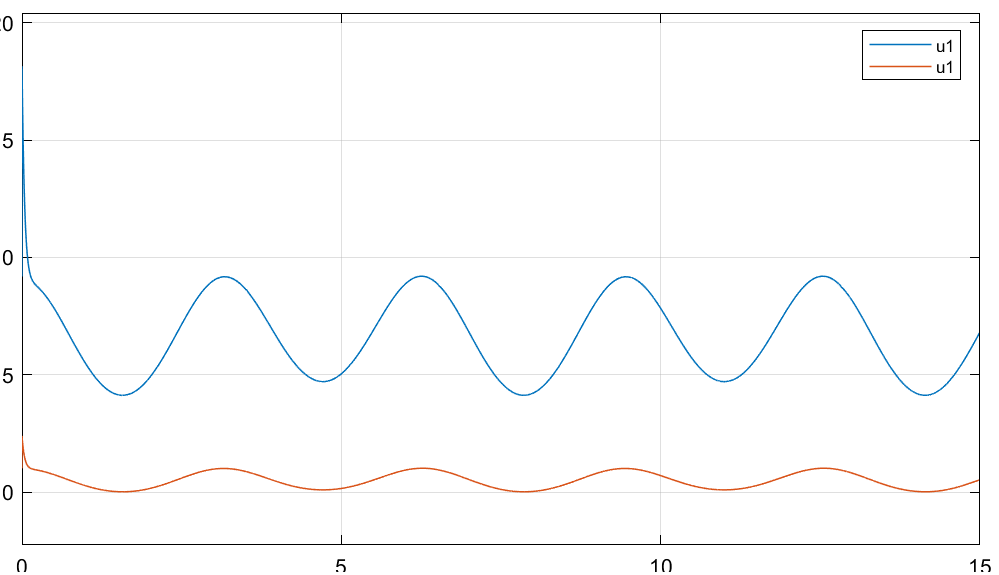


Figure 24: Pole placement+D control ideal system sine wave response - joint input torque

### Observations

* Zero Steady-state and tracking errors.
* Slight overshoot in position and big overshoot in velocity response

## PD+Gravity compensation Joint control

Table 6: PD gains for Gravity+PD control – Ideal system

|  |  |  |  |
| --- | --- | --- | --- |
| Parameter | Value | Parameter | Value |
|  | 15 |  | 3.5 |
|  | 2.5 |  | 0.5 |

### Step response

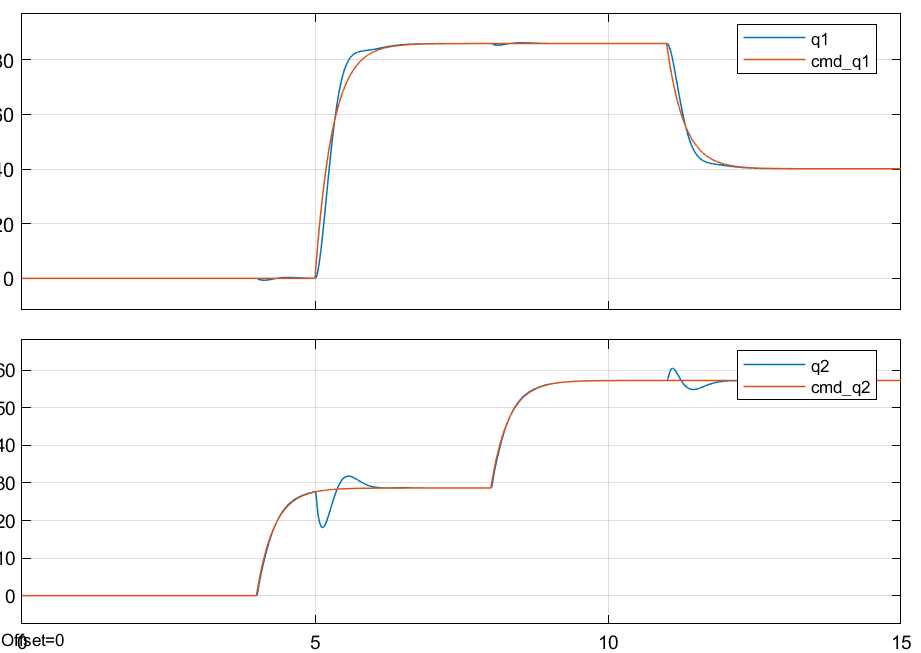


Figure 25: GRAVITY+PD control ideal system step response - joint position

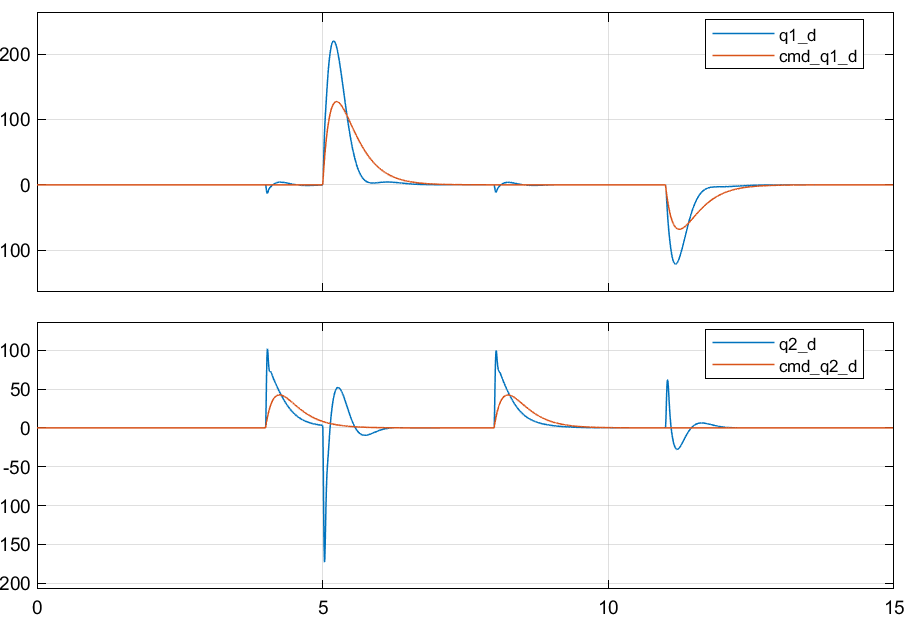


Figure 26: GRAVITY+PD control ideal system step response - joint velocity

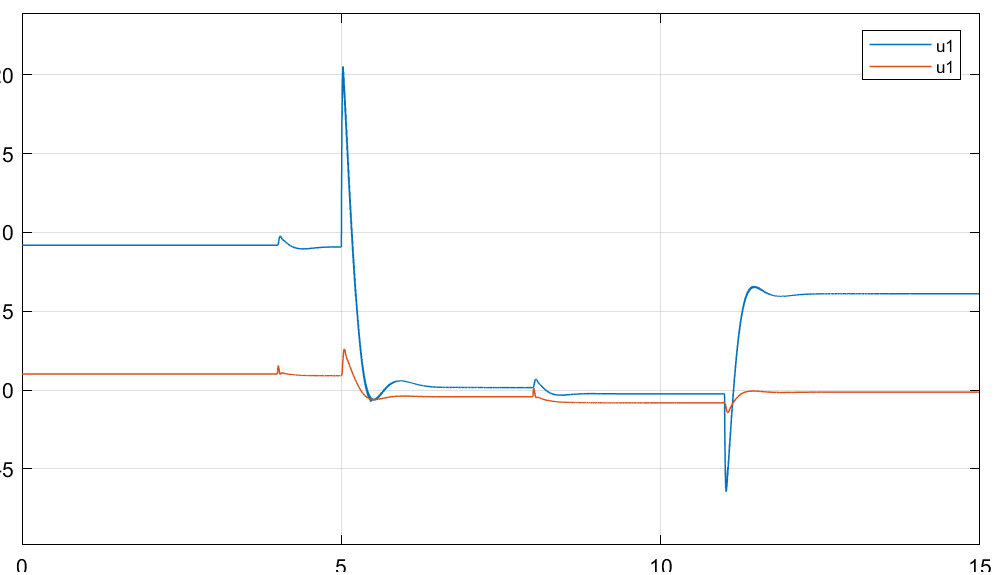


Figure 27: GRAVITY+PD control ideal system step response - joint input torque

### Sine wave response

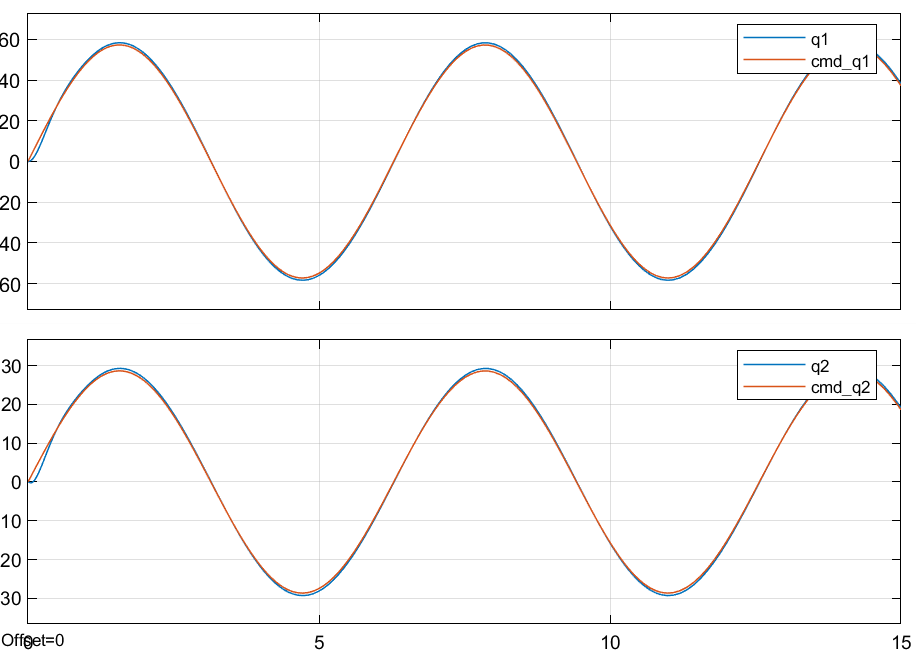


Figure 28: GRAVITY+PD control ideal system sine wave response - joint position

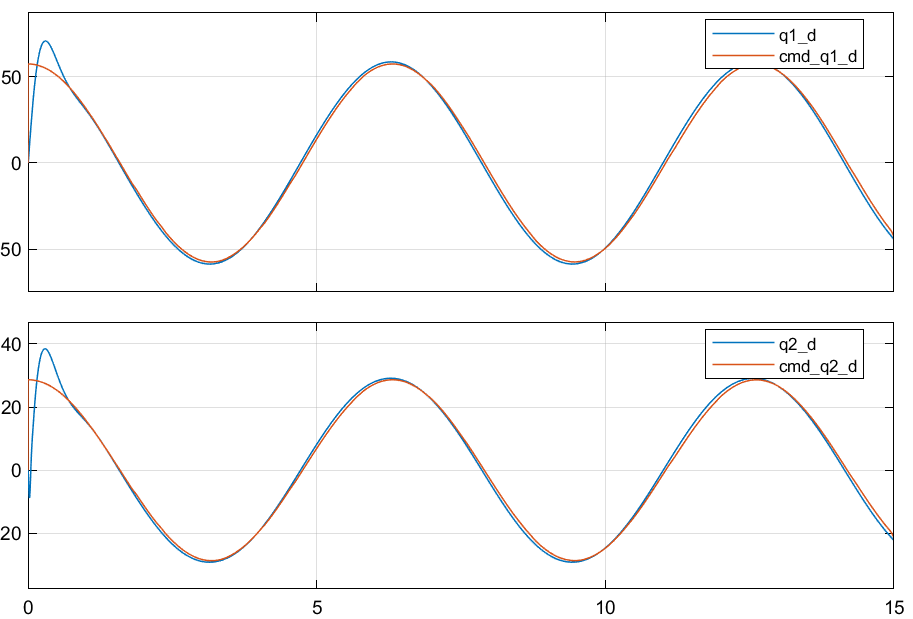


Figure 29: GRAVITY+PD control ideal system sine wave response - joint velocity

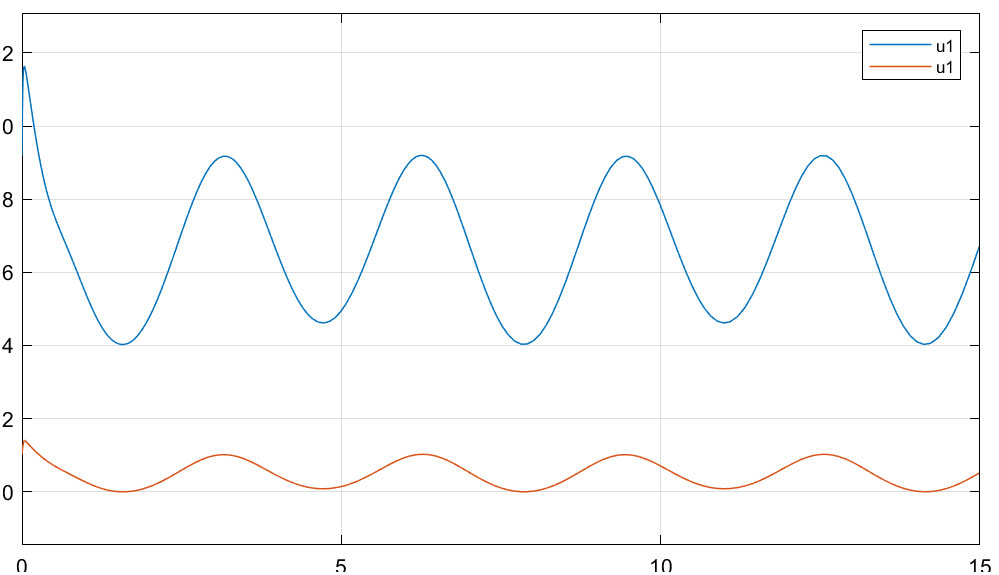


Figure 30: GRAVITY+PD control ideal system sine wave response - joint input torque

### Observations

* Steady-state converges close to zero and tracking error is within 1 degree
* Huge improvements compared to PD-only control
* Increasing PD gains will result in fast settling time and zero steady-state errors but will introduce high-frequency oscillations in commanded torques.

## Inverse dynamics control

Table 7: PD gains – Inverse control – Ideal system

|  |  |  |  |
| --- | --- | --- | --- |
| Parameter | Value | Parameter | Value |
|  | 25 |  | 50 |
|  | 12 |  | 15 |

### Step response

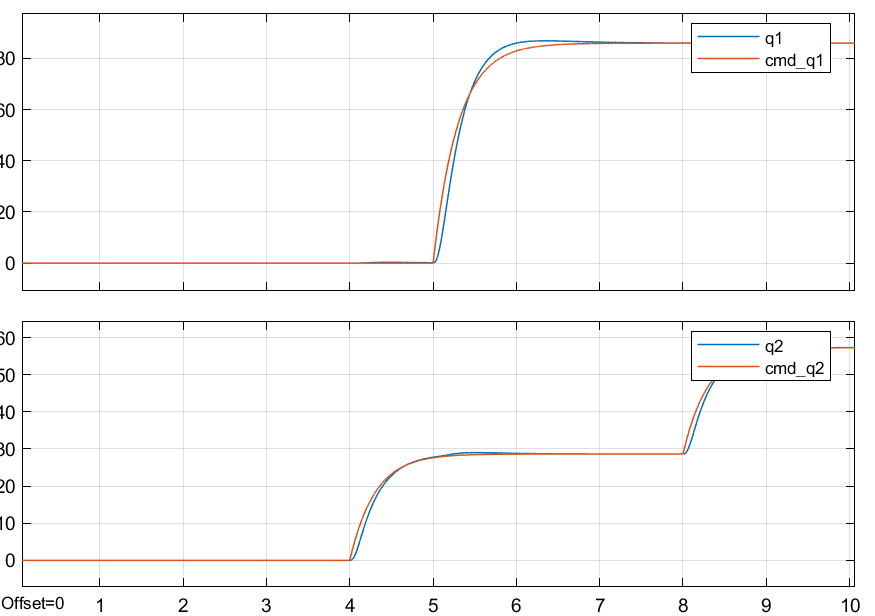


Figure 31: Inverse dynamics control ideal system step response - joint position

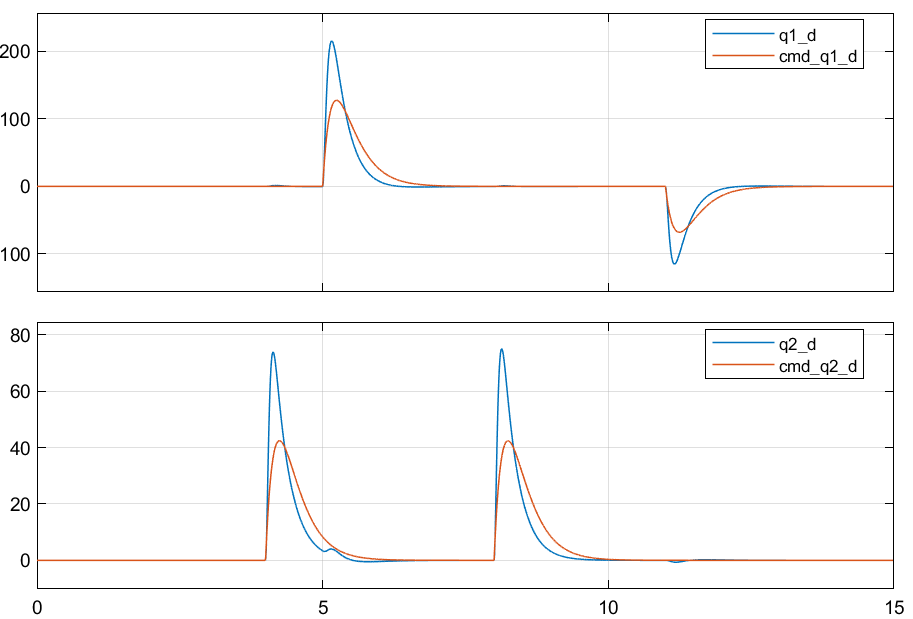


Figure 32: Inverse dynamics control ideal system step response - joint velocity

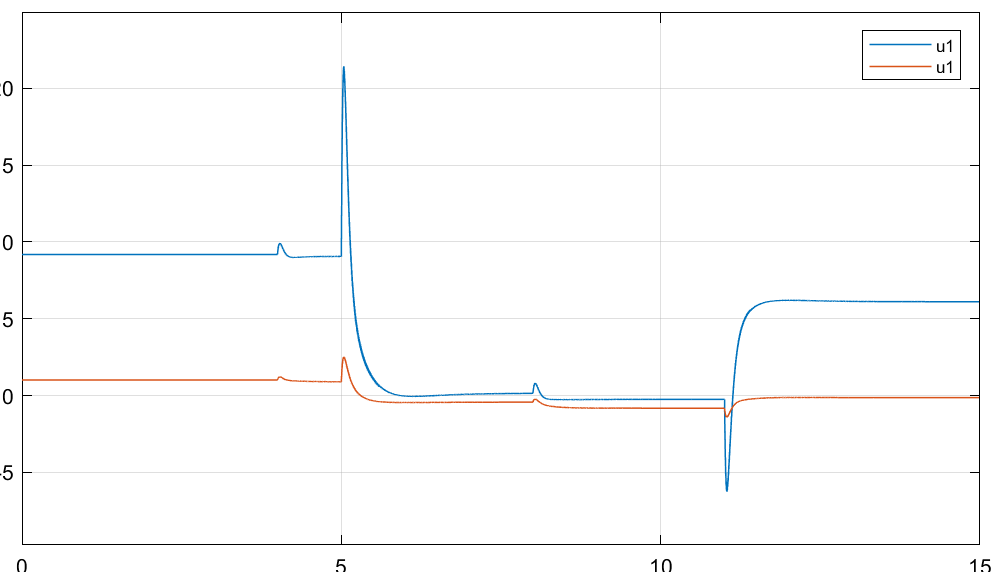


Figure 33: Inverse dynamics control ideal system step response - joint input torque

### Sine wave response

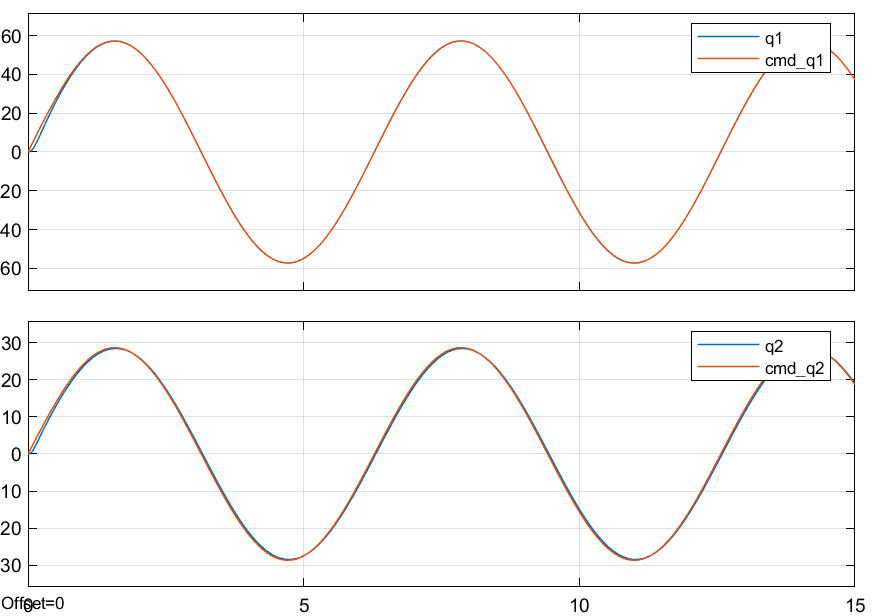


Figure 34: Inverse dynamics control ideal system sine wave response - joint position

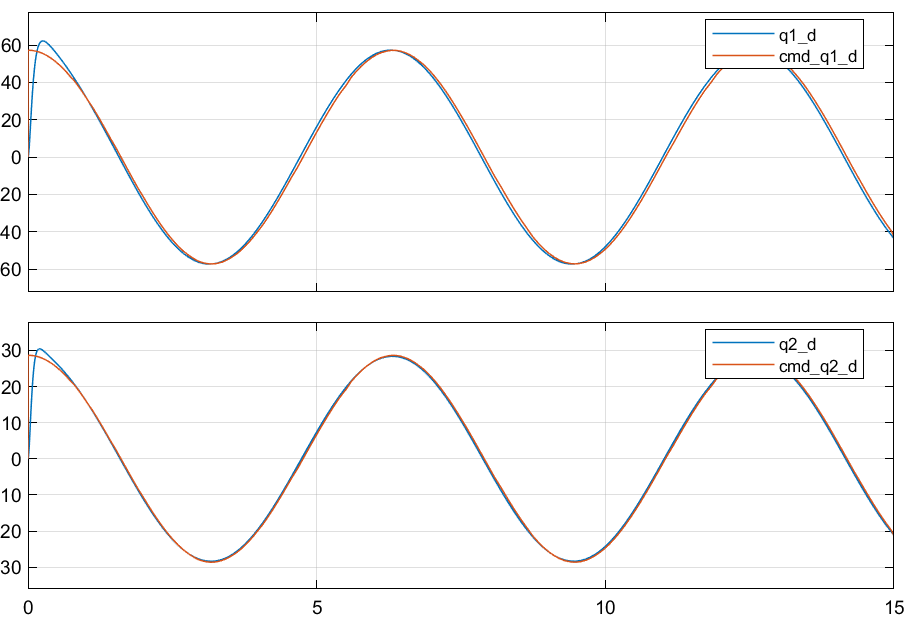


Figure 35: Inverse dynamics control ideal system sine wave response - joint velocity

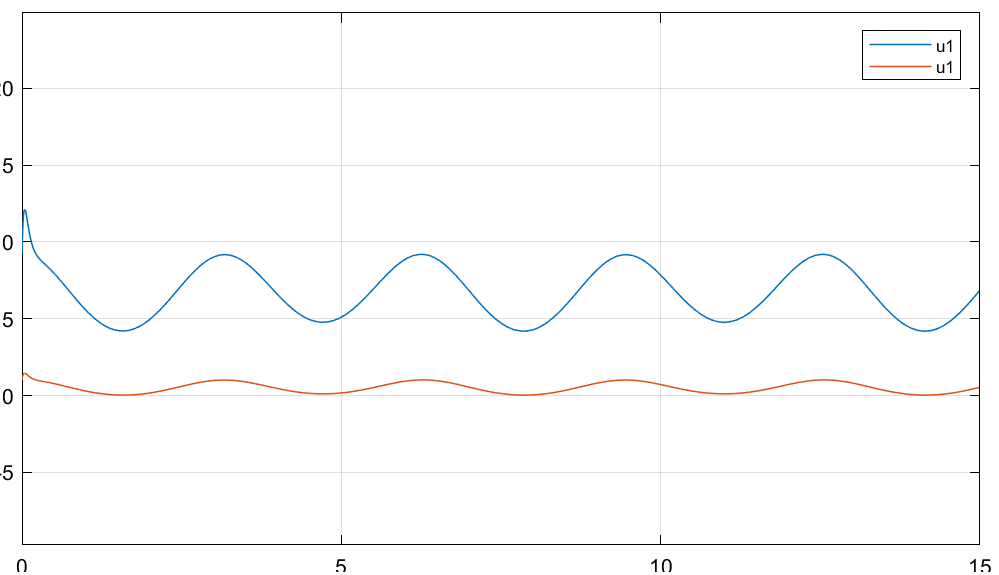


Figure 36: Inverse dynamics control ideal system sine wave response - joint input torque

### Observations

* Steady-state converges to zero and tracking error is much better than Gravity+PD control
* Huge improvements compared to PD-only control and slight improvement compared to gravity+PD control
* Increasing PD gains will result in fast settling time and zero steady-state errors but will introduce high-frequency oscillations in commanded torques.

# Simulation results (realistic) – No parameter deviation

## PD Joint control

Table 8: PD gains – realistic system

|  |  |  |  |
| --- | --- | --- | --- |
| Parameter | Value | Parameter | Value |
|  | 30 |  | 7 |
|  | 2.5 |  | 0.5 |

### Step response

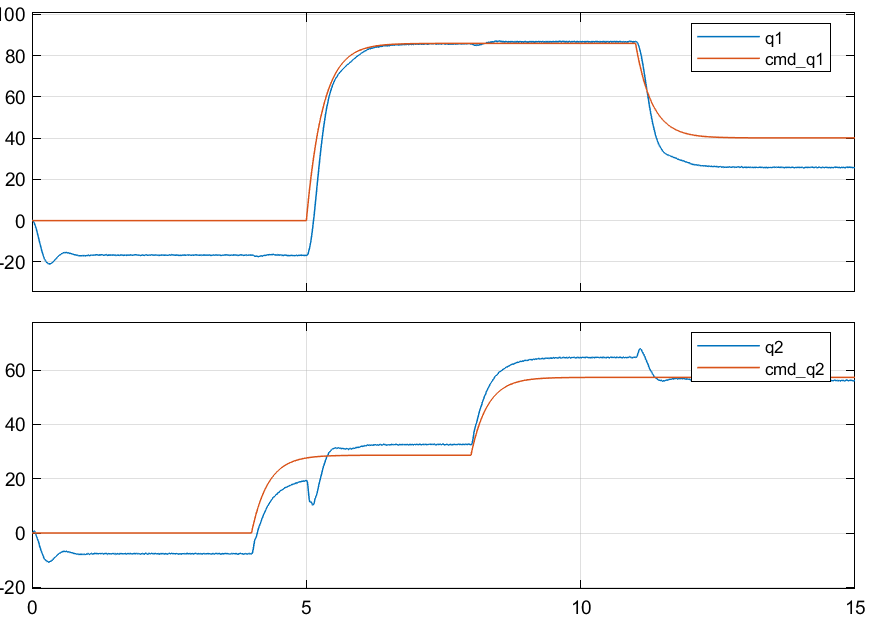


Figure 37: PD control ideal system step response - joint position

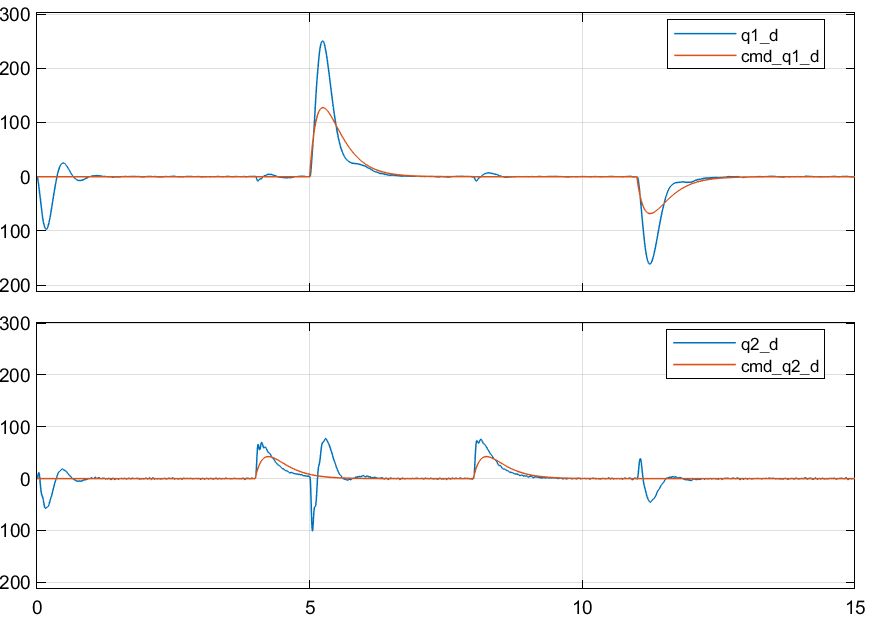


Figure 38: PD control realistic system step response - joint velocity

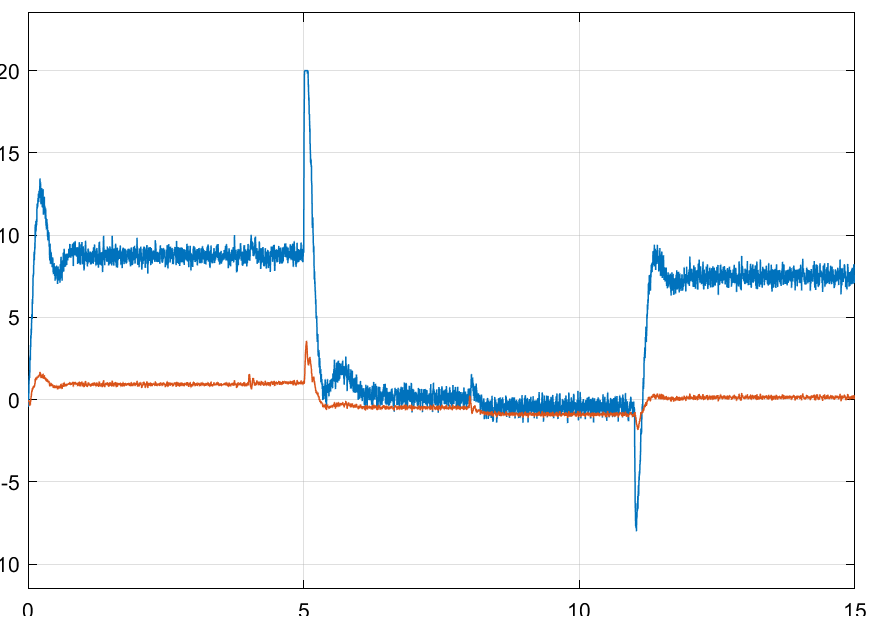


Figure 39: PD control realistic system step response - joint input torque

### Sine wave response

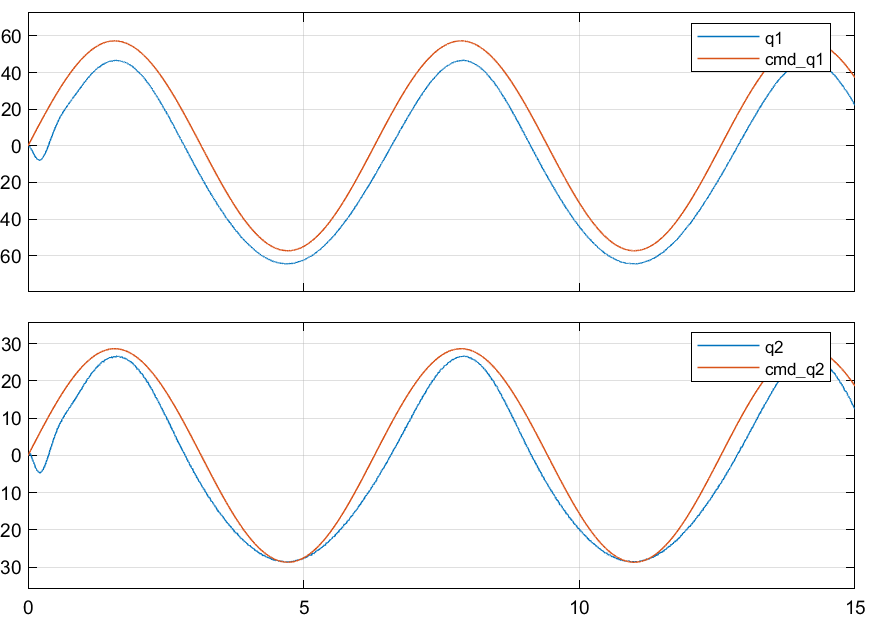


Figure 40: PD control realistic system sine wave response - joint position

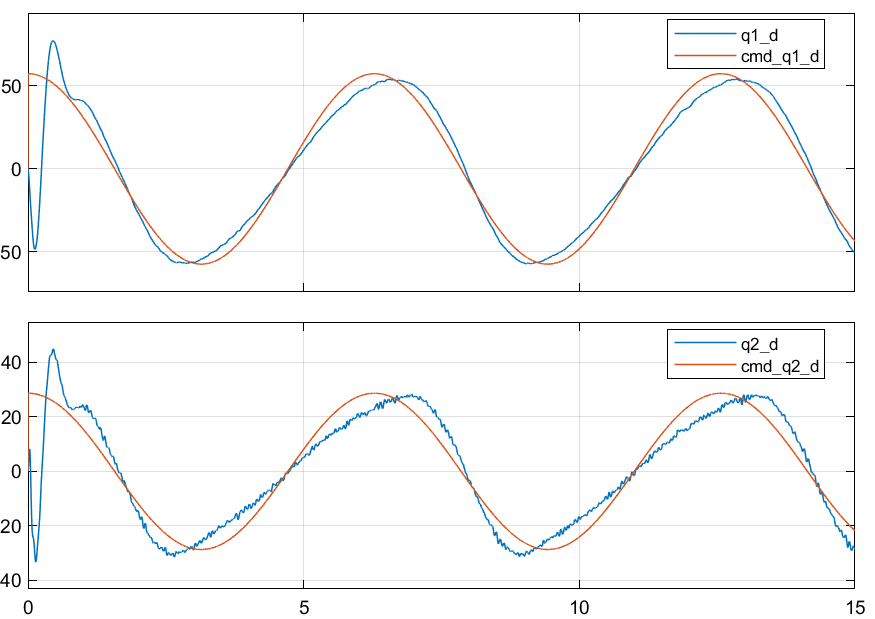


Figure 41: PD control realistic system sine wave response - joint velocity

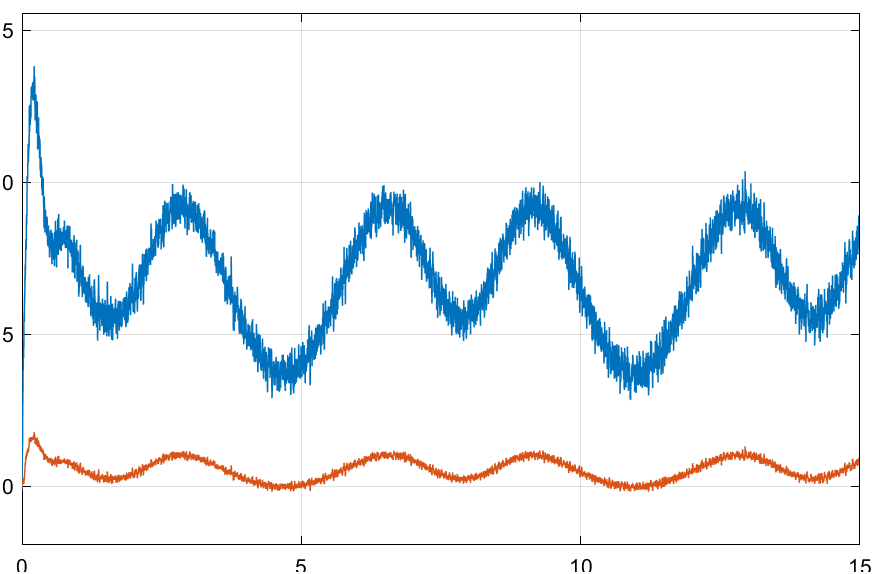


Figure 42: PD control realistic system sine wave response - joint input torque

### Observations

* Similar to ideal case in step response
* Spikes in velocity are smaller due to torque being limited
* Fluctuations in commanded torque due to measurements noise

## PID Joint control

Table 9: PID gains – realistic system

|  |  |  |  |
| --- | --- | --- | --- |
| Parameter | Value | Parameter | Value |
|  | 30 |  | 7 |
|  | 2.5 |  | 0.5 |
|  | 20 |  | 15 |

### Step response

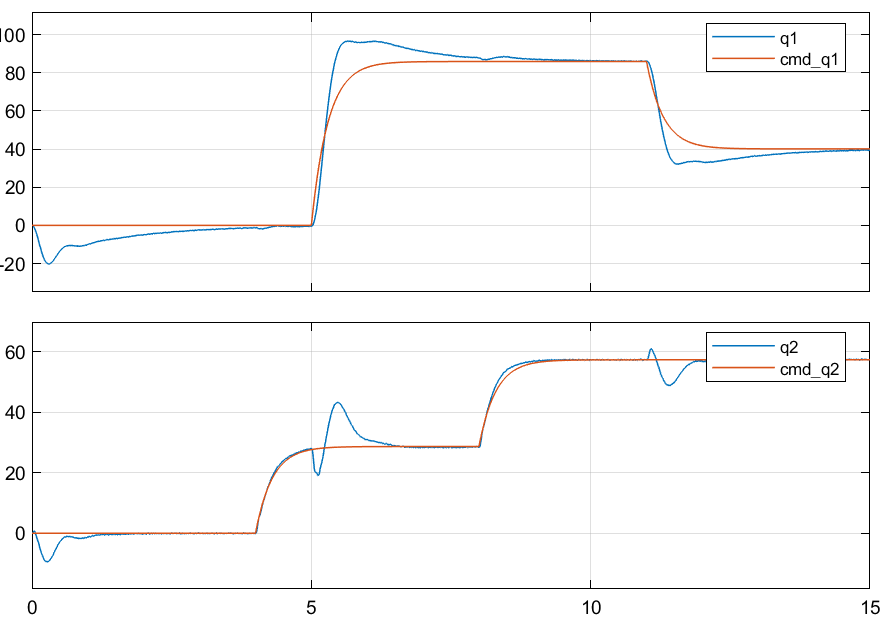


Figure 43: PID control ideal system step response - joint position

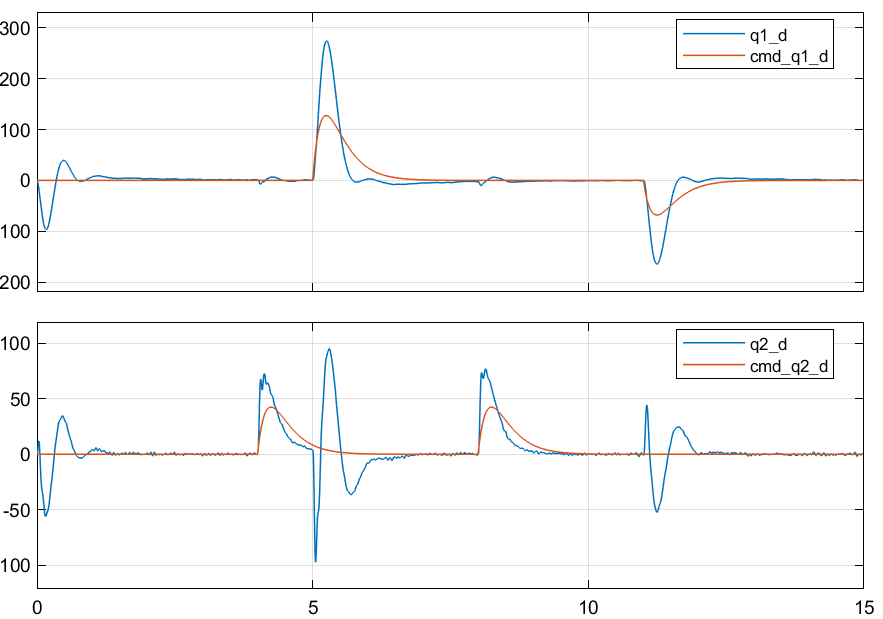


Figure 44: PID control realistic system step response - joint velocity

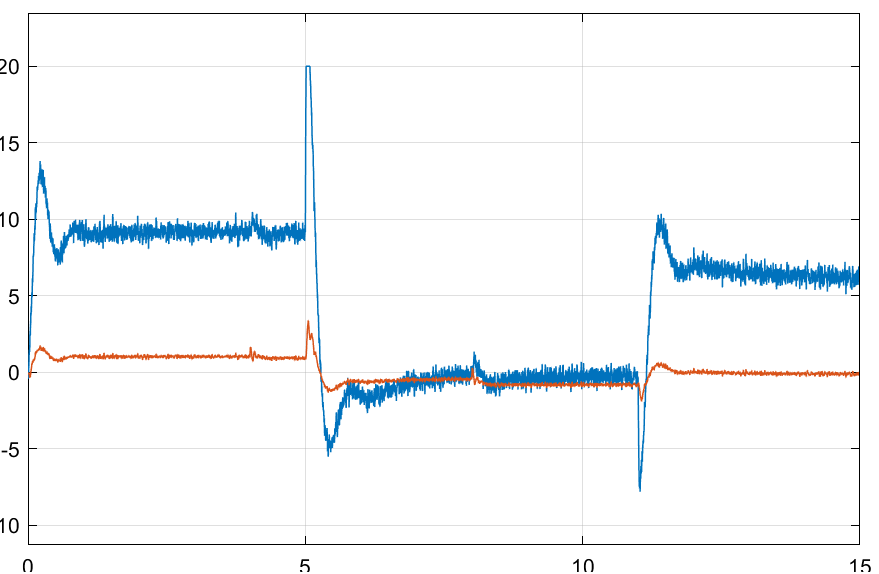


Figure 45: PID control realistic system step response - joint input torque

### Sine wave response

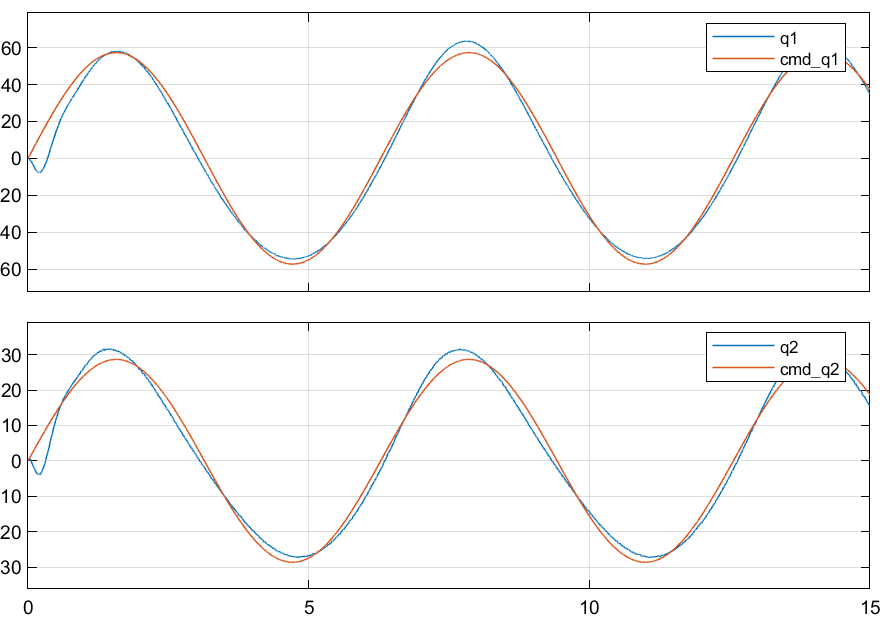


Figure 46: PID control realistic system sine wave response - joint position

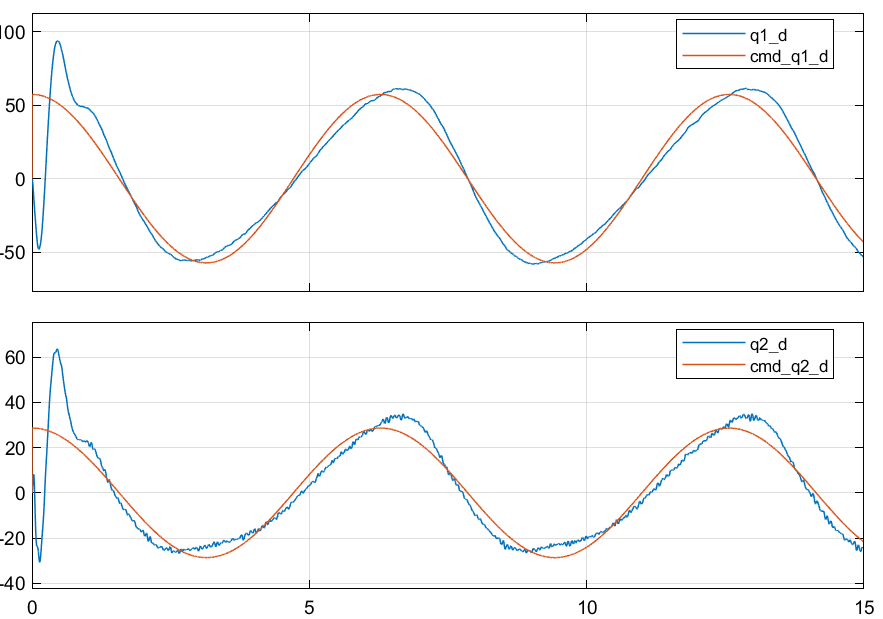


Figure 47: PID control realistic system sine wave response - joint velocity

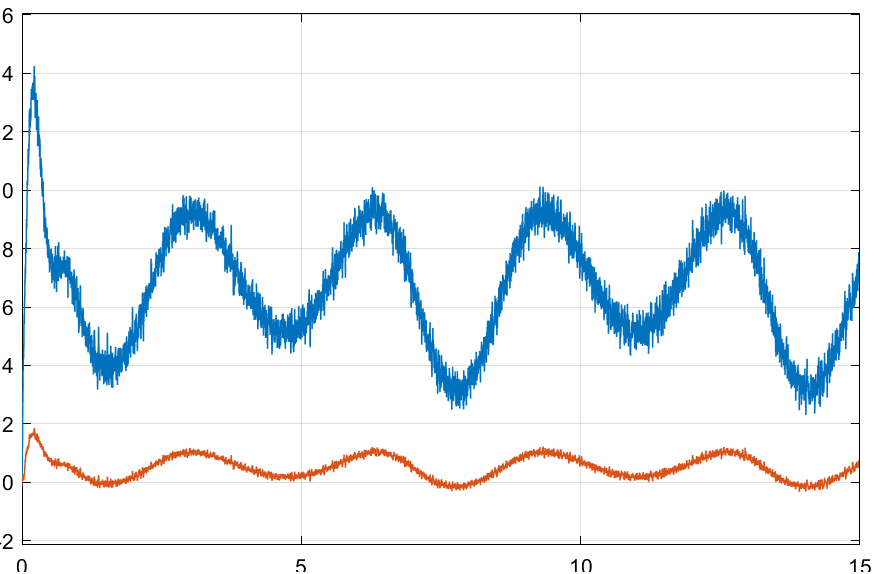


Figure 48: PID control realistic system sine wave response - joint input torque

### Observations

* Steady-state goes to zero if given enough time
* Tracking error is still not zero due to response lag
* Performance is worse than ideal case in terms of overshoot and settling time
* Has small-magnitude ripples in commanded torque but no high-magnitude high frequency damped oscillations similar to that of ideal case
* Torque is bounded unlike ideal case and huge spikes in commanded torque happen
* Gains can be further tuned and if increased a lot, tracking and steady-state errors will be smaller but it will result in huge oscillations in commanded torques.

## Pole Placement Control

Table 10: Pole values and feedback gain- realistic case

|  |  |
| --- | --- |
| Parameter | Value |
| poles | [-25, -24, -23, -5.8] |
|  |  |

### Step response

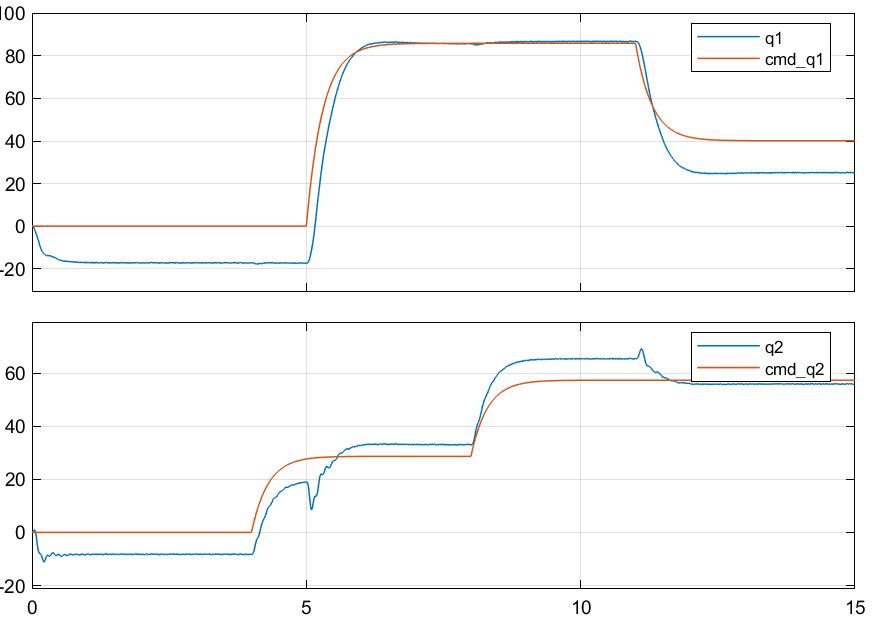


Figure 49: Pole placement control realistic system step response - joint position

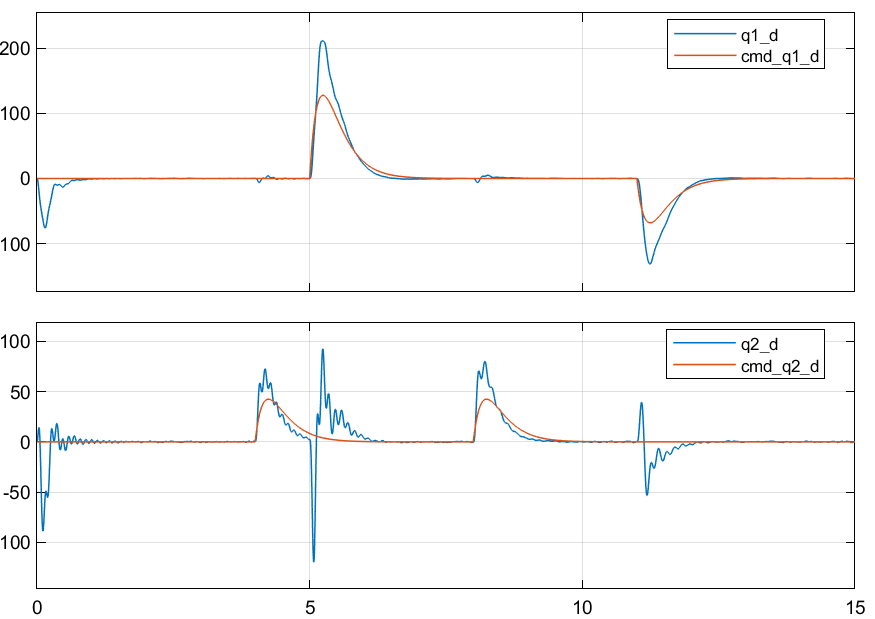


Figure 50: Pole placement control realistic system step response - joint velocity

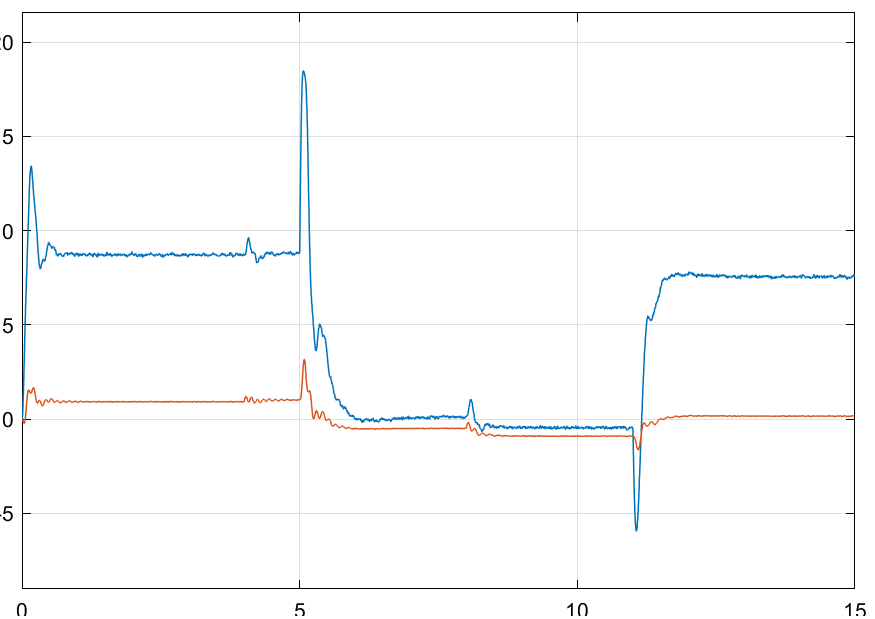


Figure 51: Pole placement control realistic system step response - joint input torque

### Sine wave response

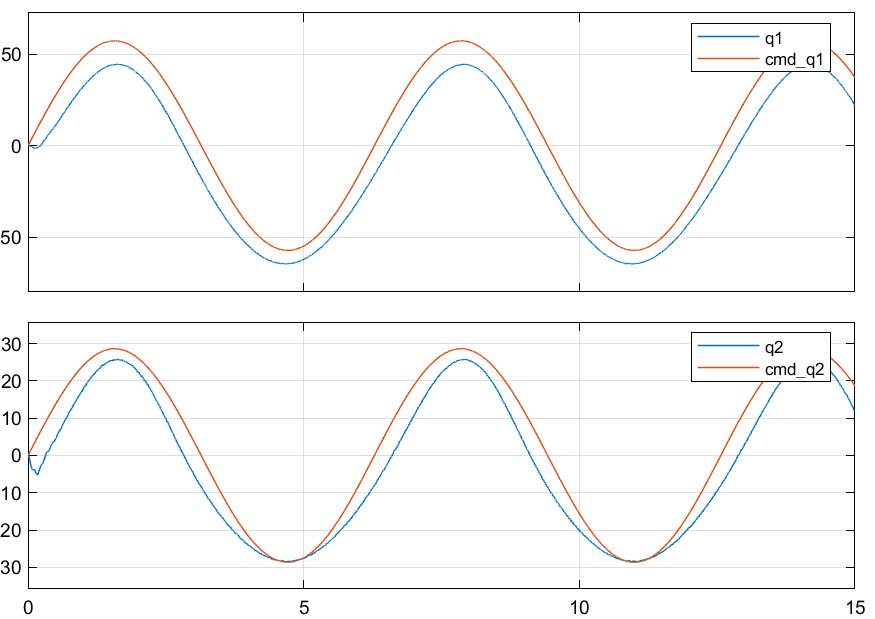


Figure 52: Pole placement control realistic system sine wave response - joint position

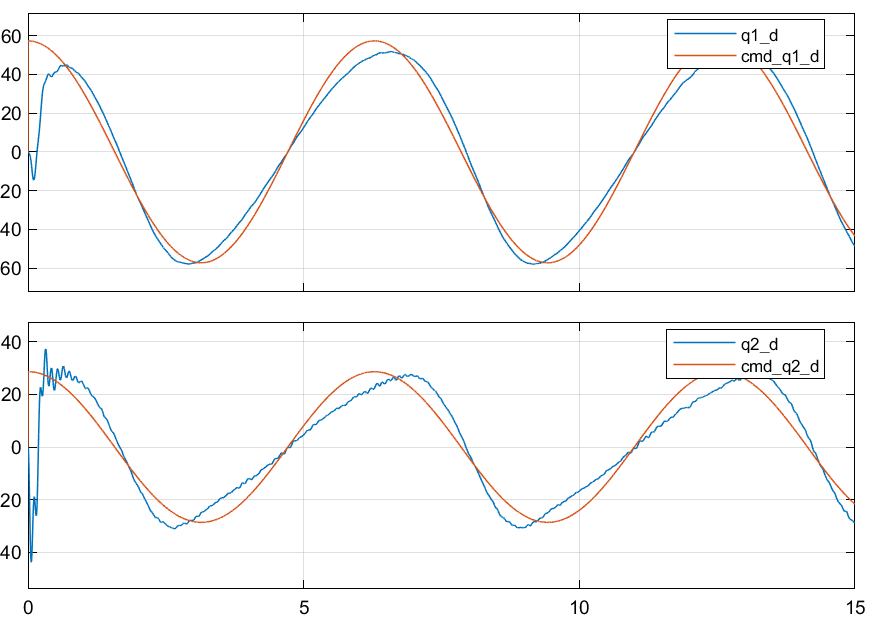


Figure 53: Pole placement control realistic system sine wave response - joint velocity

### Observations

* Nonzero Steady-state and tracking error

## Pole Placement Control with feedforward disturbance terms

### Step response

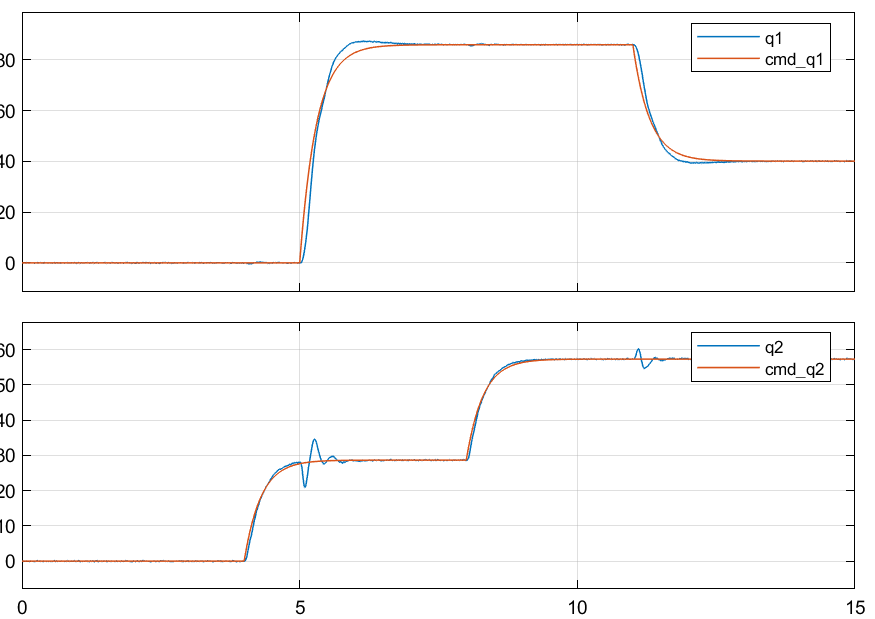


Figure 54: Pole placement+D control realistic system step response - joint position

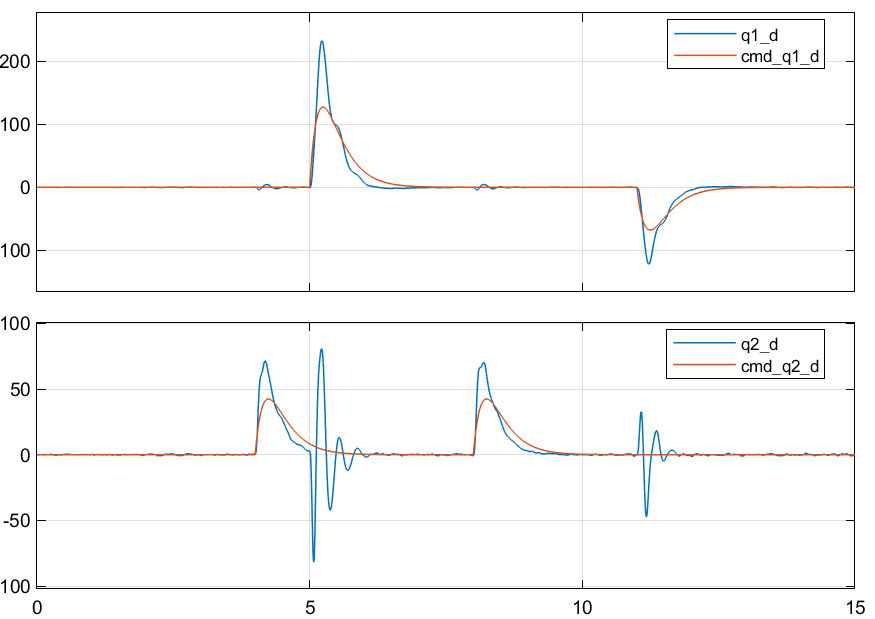
****

Figure 55: Pole placement+D control realistic system step response - joint velocity

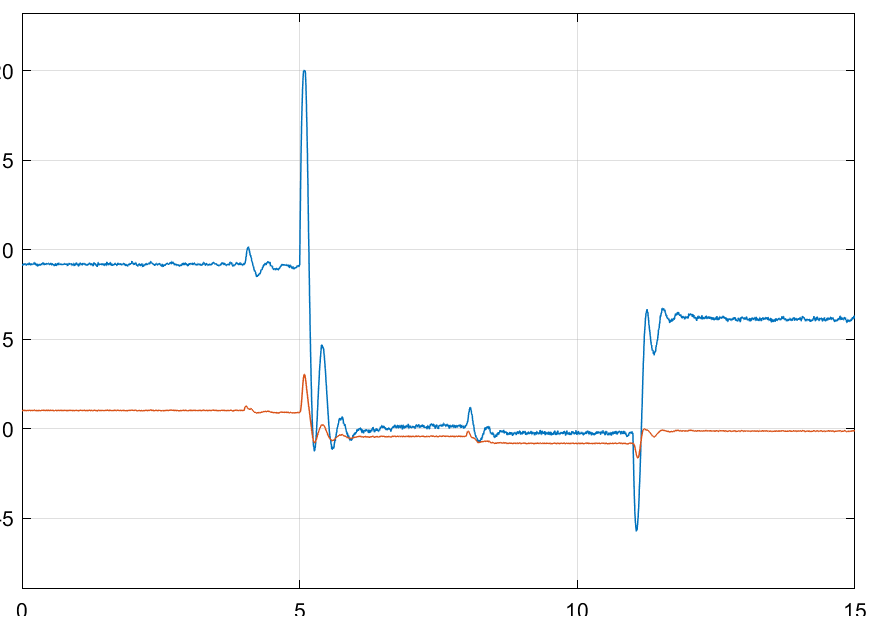


Figure 56: Pole placement+D control realistic system step response - joint input torque

### Sine wave response

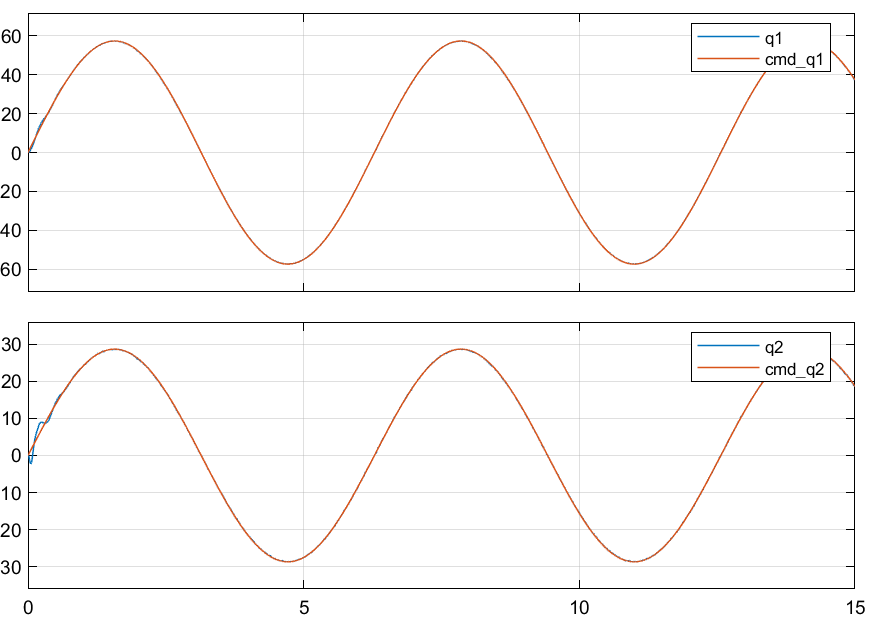


Figure 57: Pole placement+D control realistic system sine wave response - joint position

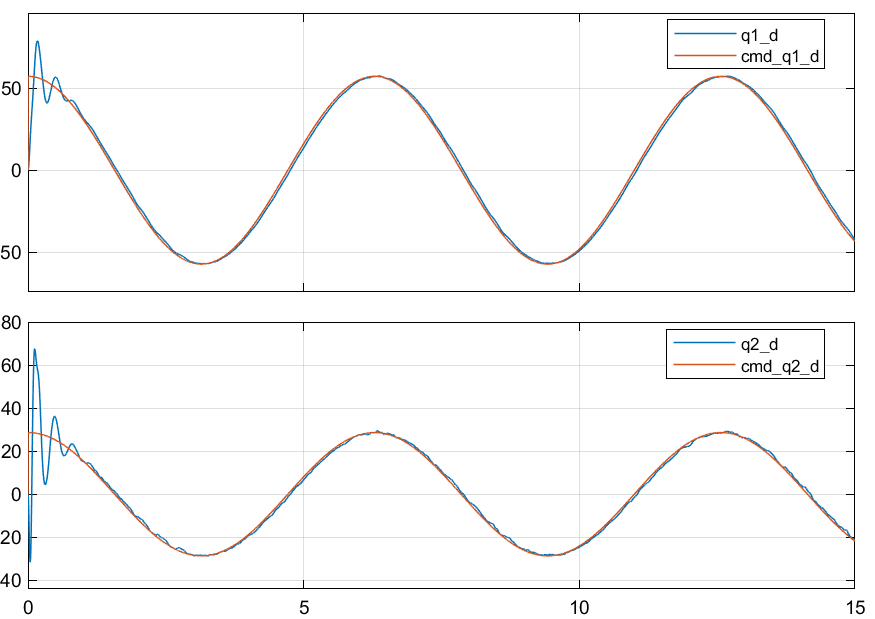


Figure 58: Pole placement+D control realistic system sine wave response - joint velocity

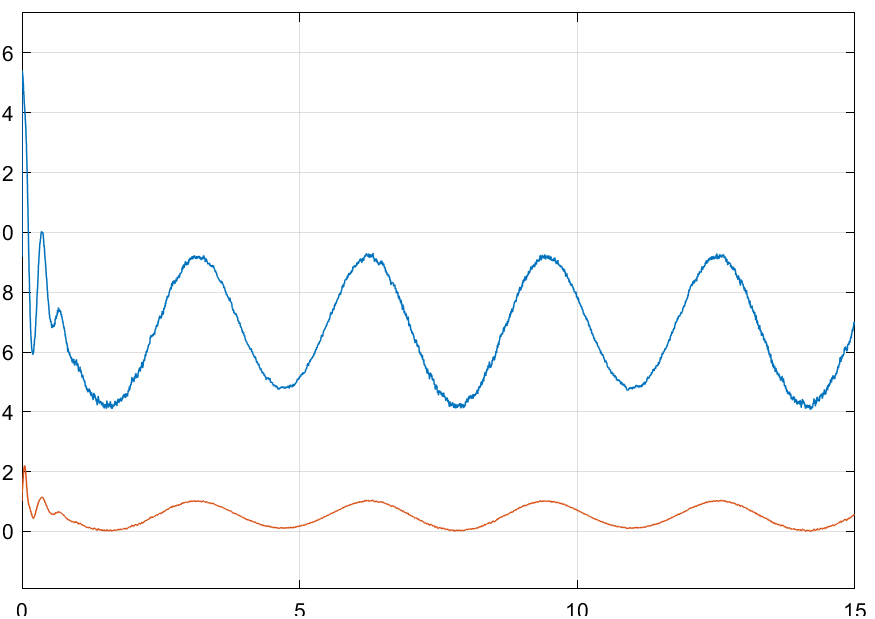


Figure 59: Pole placement+D control realistic system sine wave response - joint input torque

### Observations

* Zero Steady-state and tracking errors.
* Some overshoot in position and big overshoot in velocity step response
* Tracking the sine wave position and velocity is quite good

## PD+Gravity compensation Joint control

Table 11: PD gains for Gravity+PD control – realistic system

|  |  |  |  |
| --- | --- | --- | --- |
| Parameter | Value | Parameter | Value |
|  | 15 |  | 3.5 |
|  | 2.5 |  | 0.5 |

### Step response

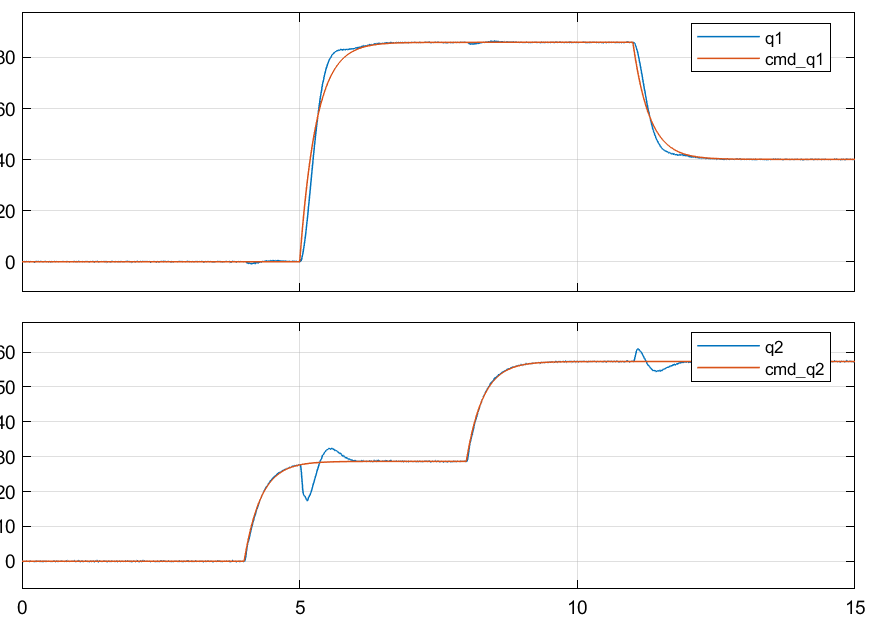


Figure 60: GRAVITY+PD control realistic system step response - joint position

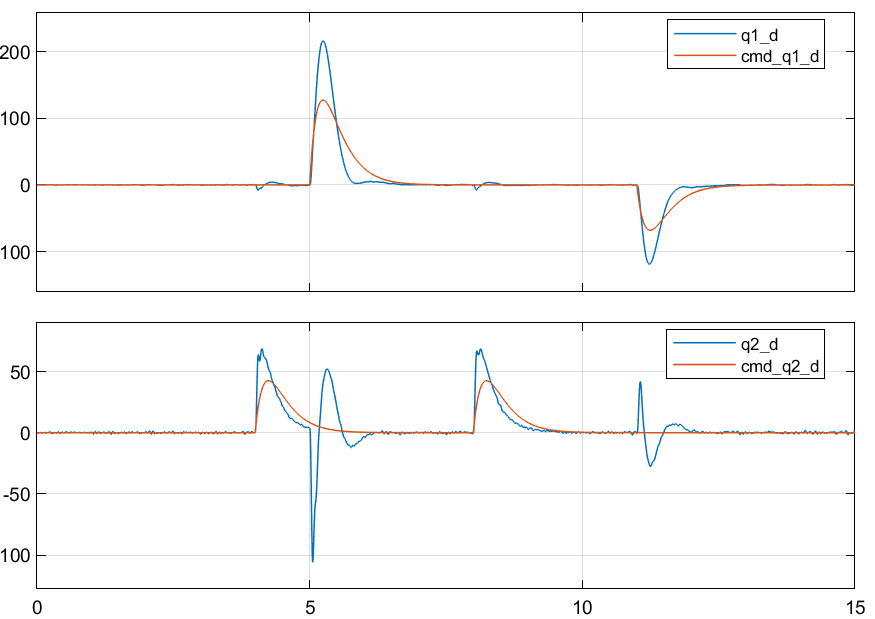


Figure 61: GRAVITY+PD control realistic system step response - joint velocity

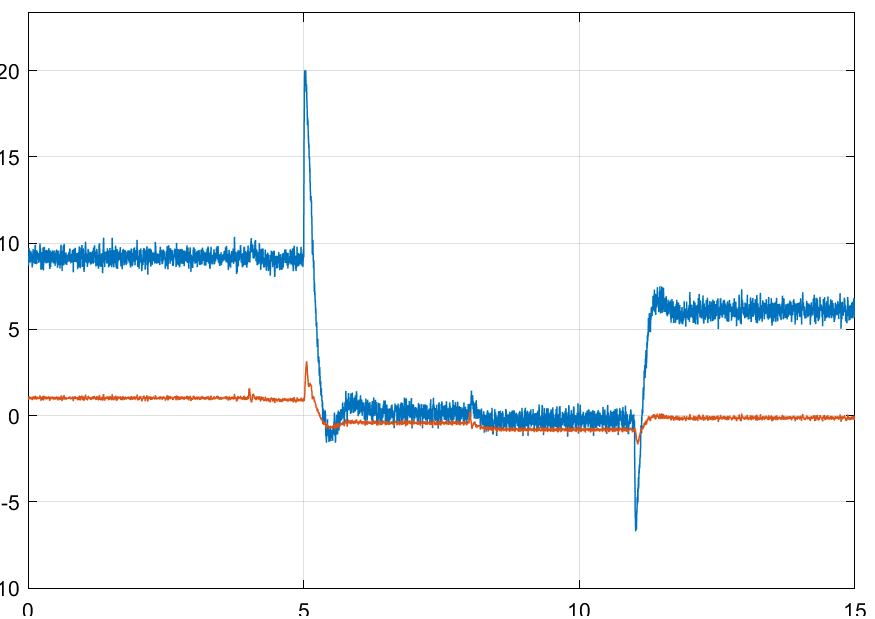


Figure 62: GRAVITY+PD control realistic system step response - joint input torque

### Sine wave response

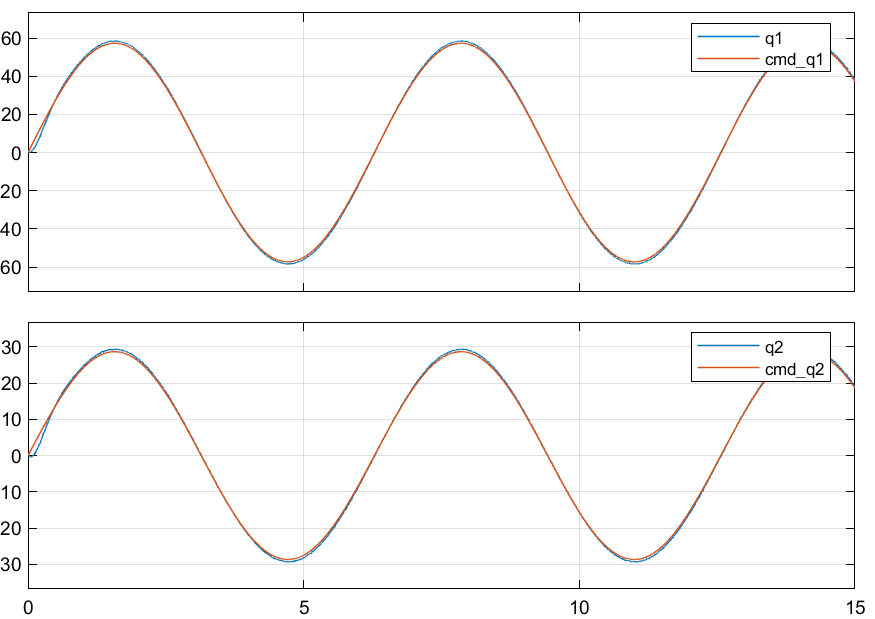


Figure 63: GRAVITY+PD control realistic system sine wave response - joint position

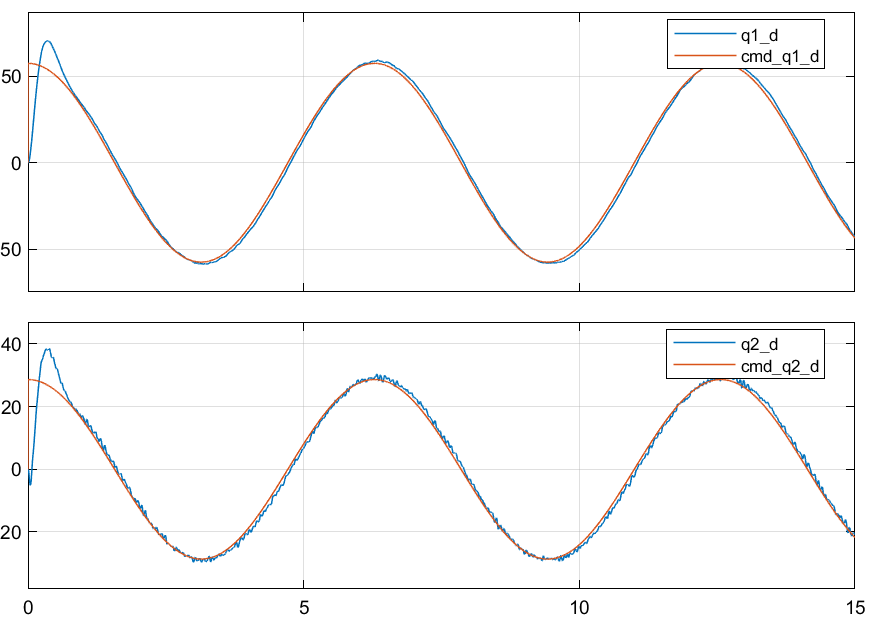


Figure 64: GRAVITY+PD control realistic system sine wave response - joint velocity

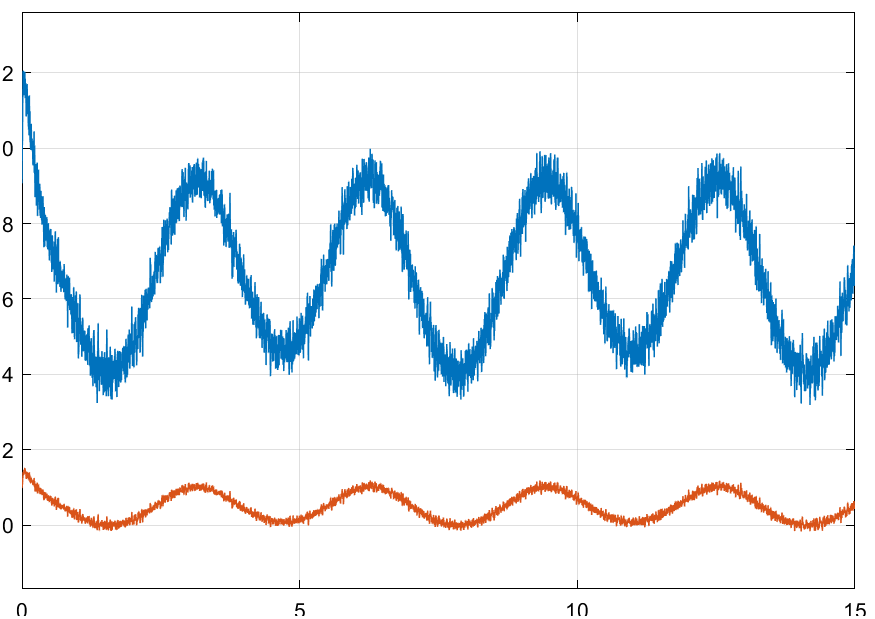


Figure 65: GRAVITY+PD control realistic system sine wave response - joint input torque

### Observations

* Steady-state converges to zero and tracking error is within 1 degree
* Similar to ideal case response
* Increasing PD gains will result in faster settling time and zero steady-state errors but will introduce high-frequency oscillations in commanded torques.

## Inverse dynamics control

Table 12: PD gains – Inverse control – Realistic system

|  |  |  |  |
| --- | --- | --- | --- |
| Parameter | Value | Parameter | Value |
|  | 25 |  | 50 |
|  | 12 |  | 15 |

### Step response

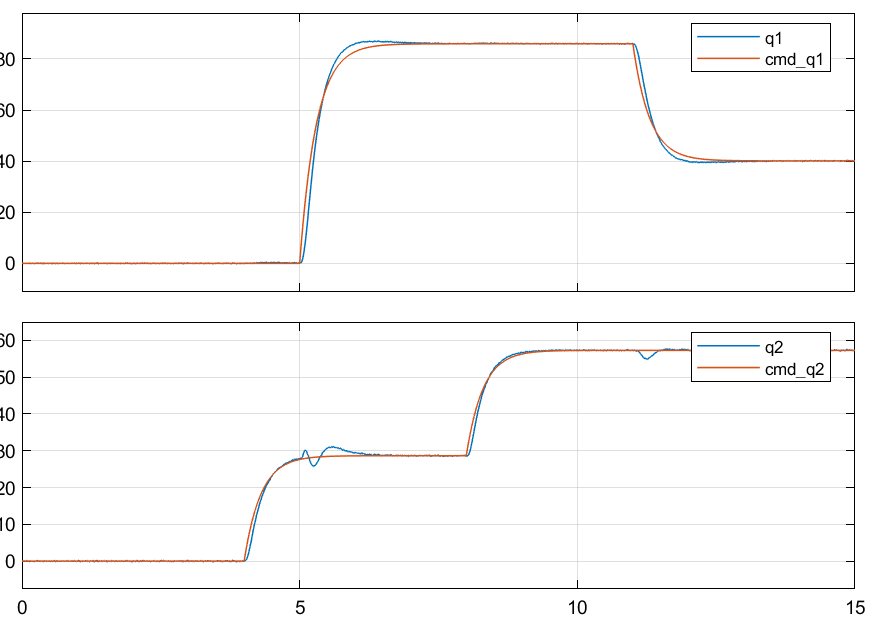


Figure 66: Inverse dynamics control realistic system step response - joint position

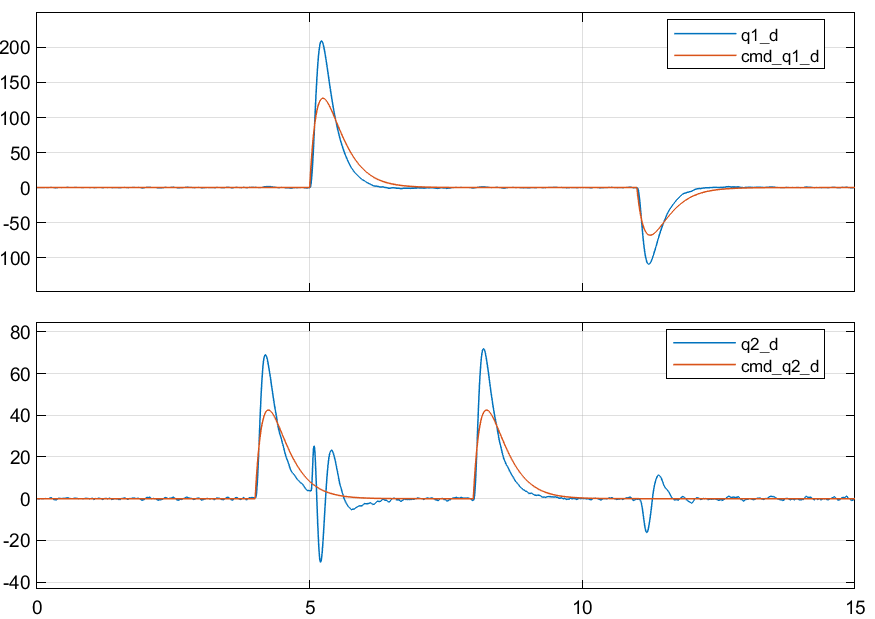


Figure 67: Inverse dynamics control realistic system step response - joint velocity

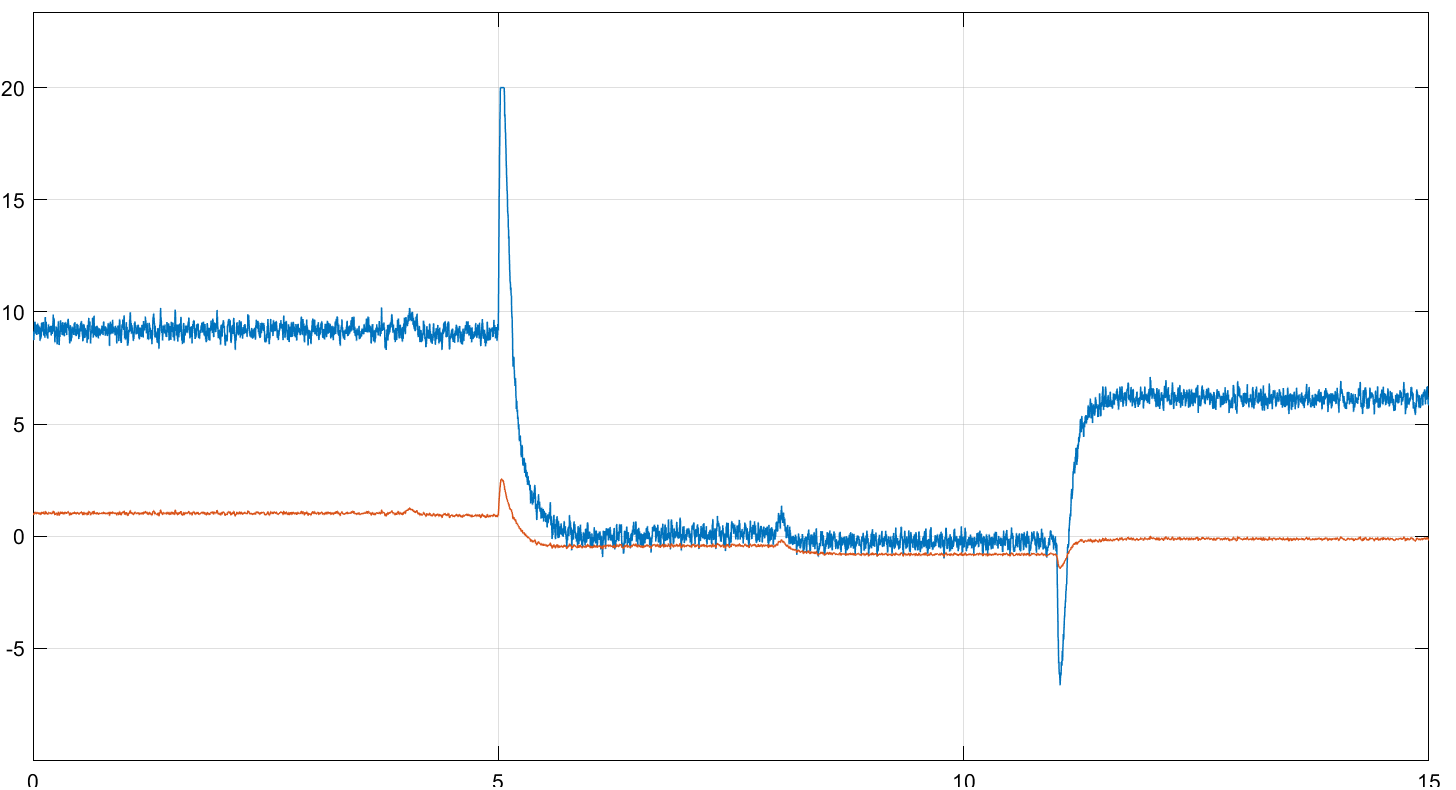


Figure 68: Inverse dynamics control realistic system step response - joint input torque

### Sine wave response

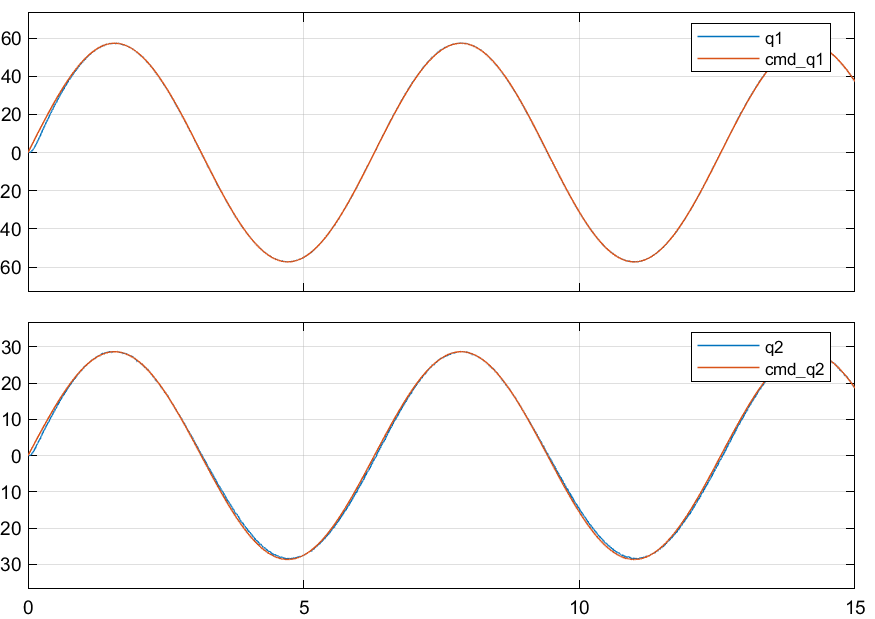


Figure 69: Inverse dynamics control realistic system sine wave response - joint position

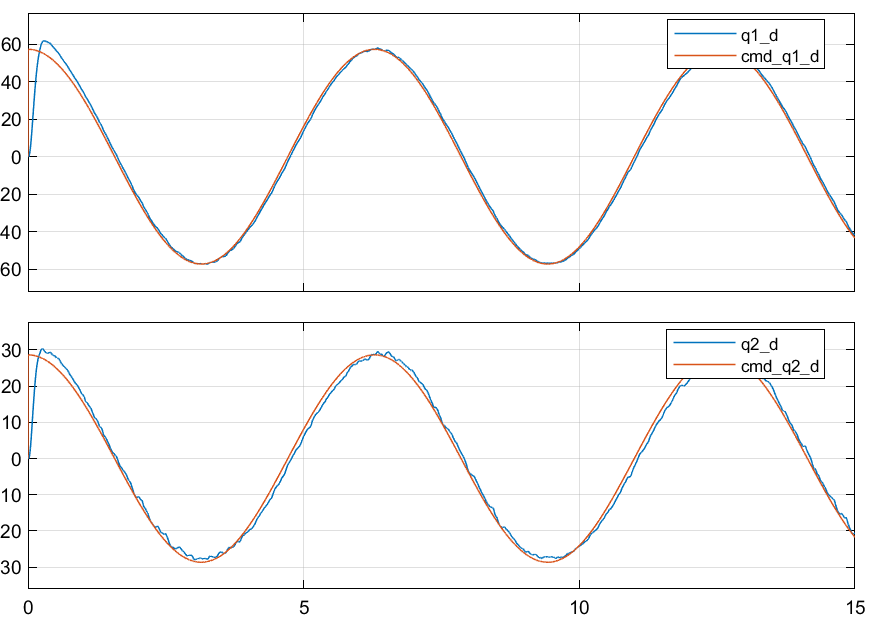


Figure 70: Inverse dynamics control realistic system sine wave response - joint velocity

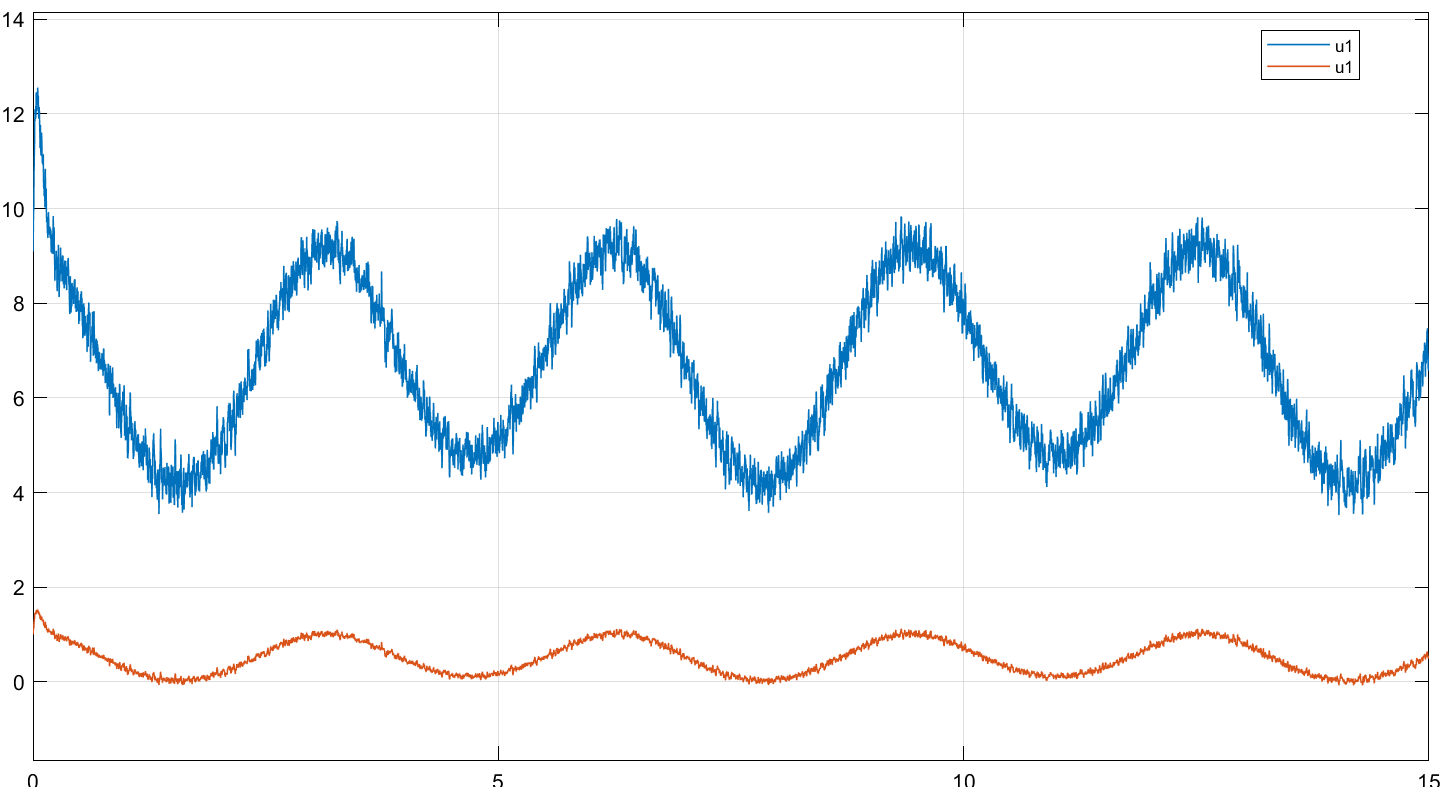


Figure 71: Inverse dynamics control realistic system sine wave response - joint input torque

### Observations

* Steady-state converges to zero and tracking error is better than Gravity+PD control
* Similar to ideal case with some ripples in velocity response due to measurement noise.
* Increasing PD gains will result in fast settling time and zero steady-state errors but will introduce high-frequency oscillations in commanded torques.

## Robust control

Table 13: PD gains for robust control – Realistic system

|  |  |  |  |
| --- | --- | --- | --- |
| Parameter | Value | Parameter | Value |
|  | 25 |  | 45 |
|  | 7 |  | 35 |
|  | 0.1 |  | 1 |
|  |  |  |  |

### Step response

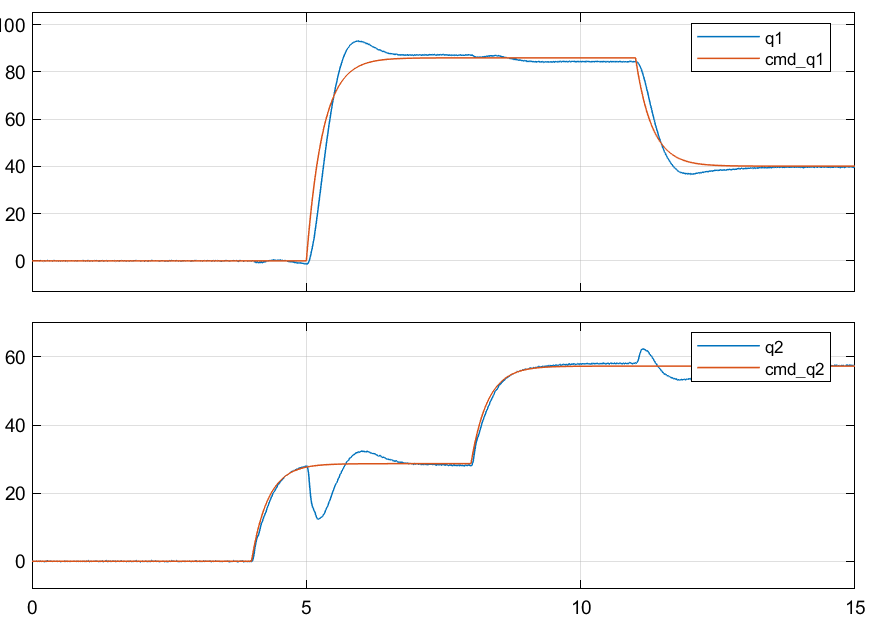


Figure 72: Robust control realistic system step response - joint position

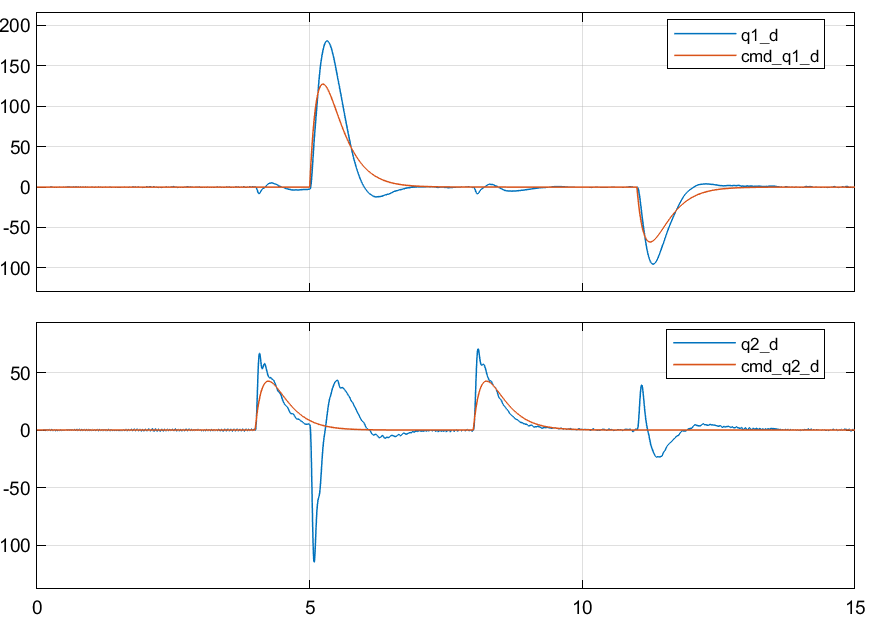


Figure 73: Robust control realistic system step response - joint velocity

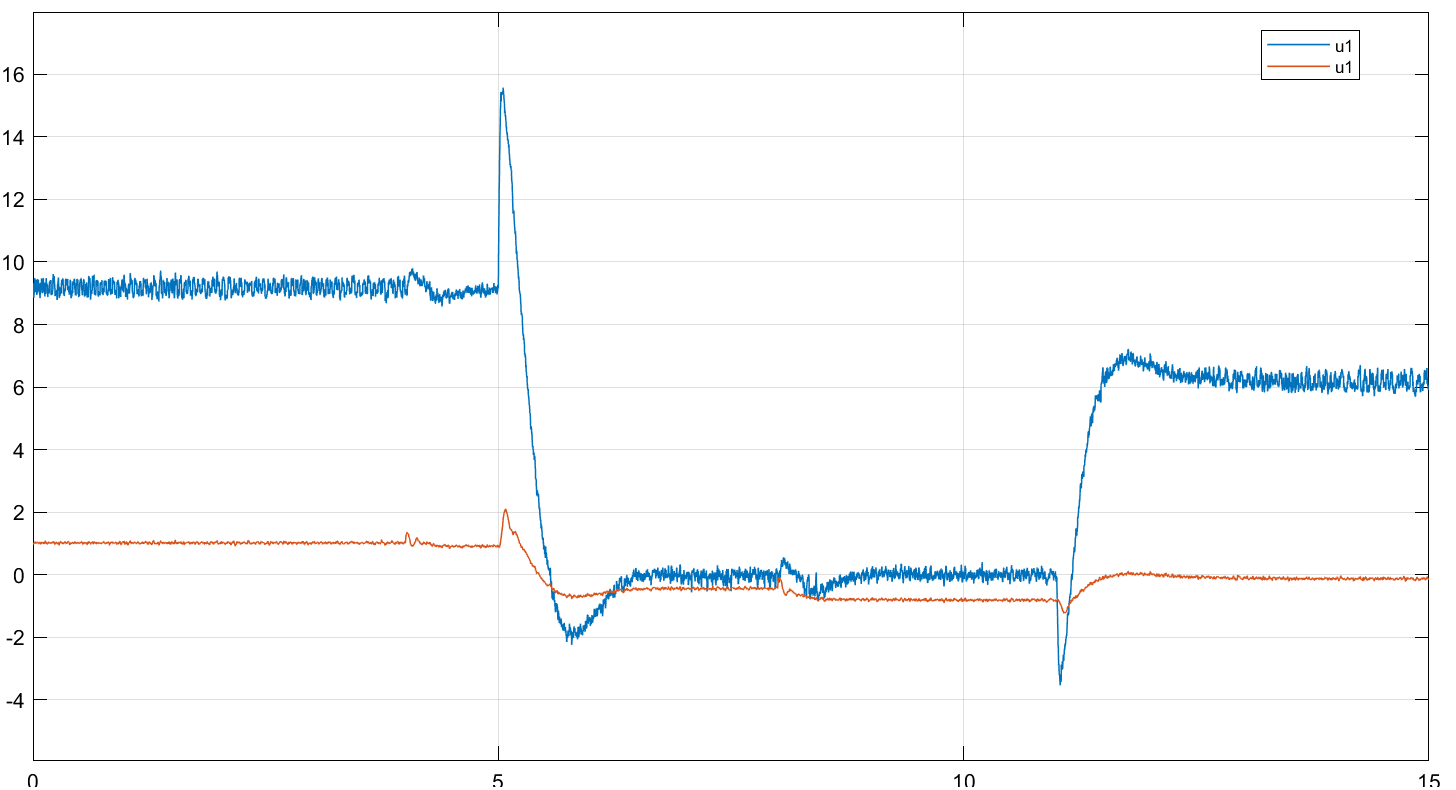


Figure 74: Robust control realistic system step response - joint input torque

### Sine wave response

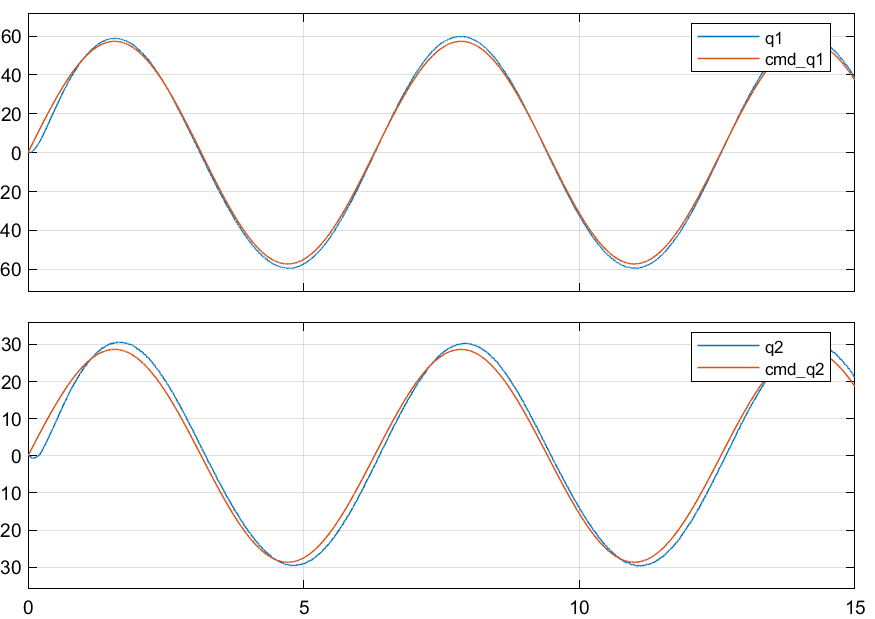


Figure 75: Robust control realistic system sine wave response - joint position

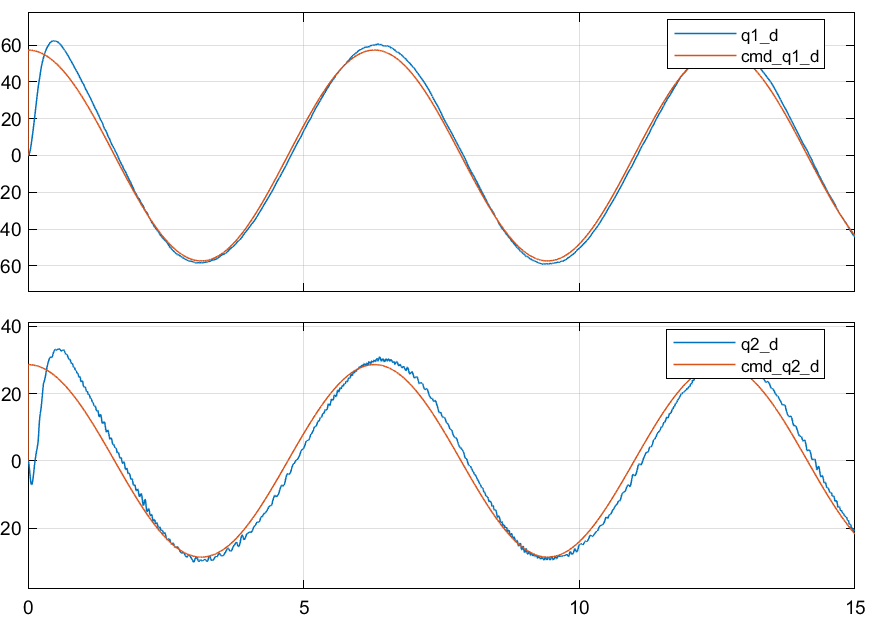


Figure 76: Robust control realistic system sine wave response - joint velocity

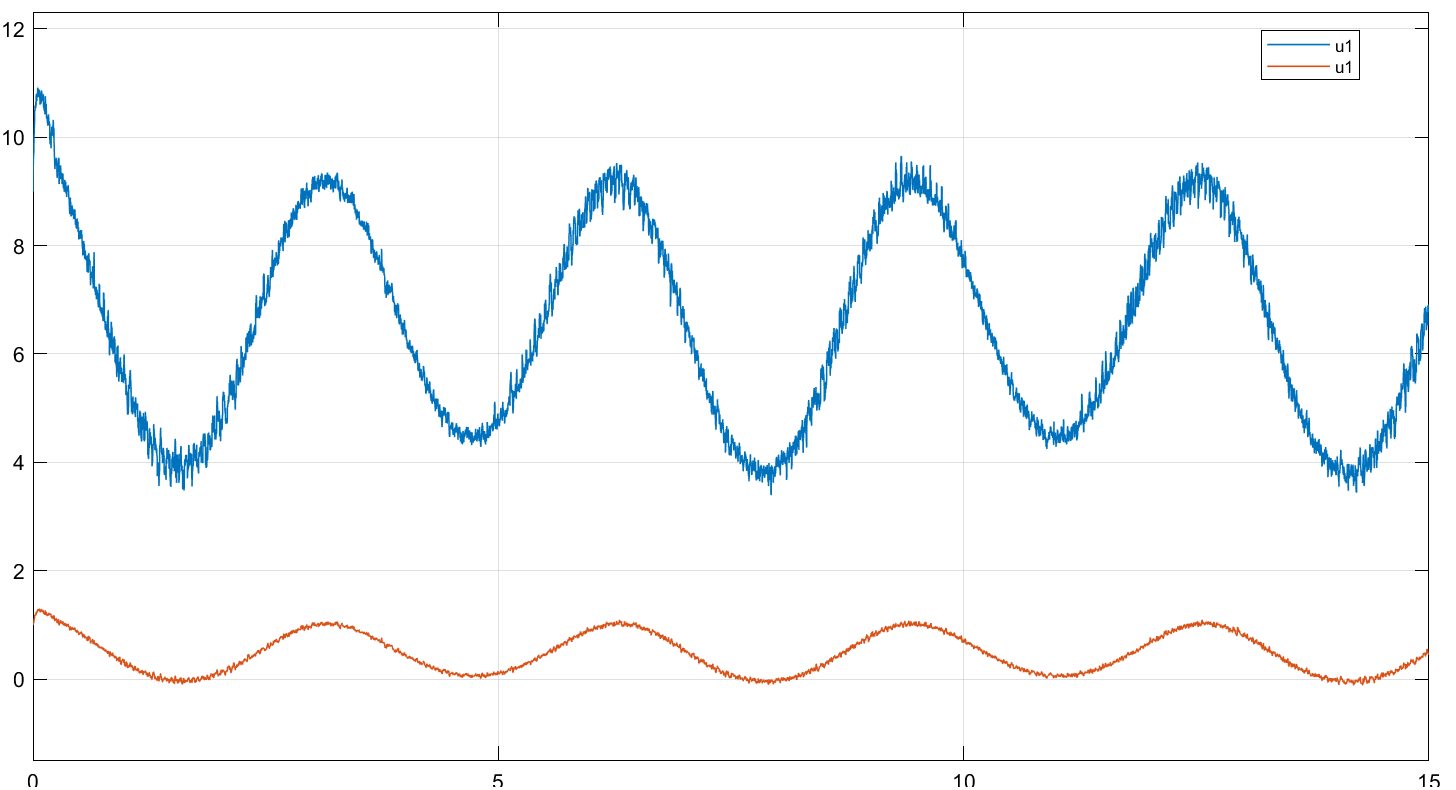


Figure 77: Robust control realistic system sine wave response - joint input torque

### Observations

* Worse in performance than inverse dynamics control and gravity+PD control

# Simulation results (realistic) – with parameter deviation

## Pole Placement Control with feedforward disturbance terms

### Step response

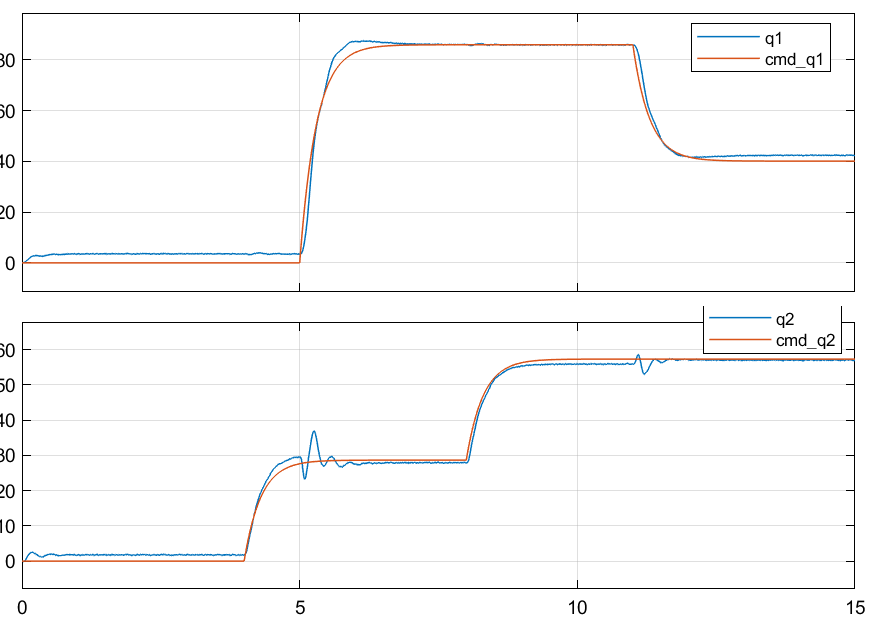


Figure 78: Pole placement+D control realistic system step response - joint position – with parameter deviation

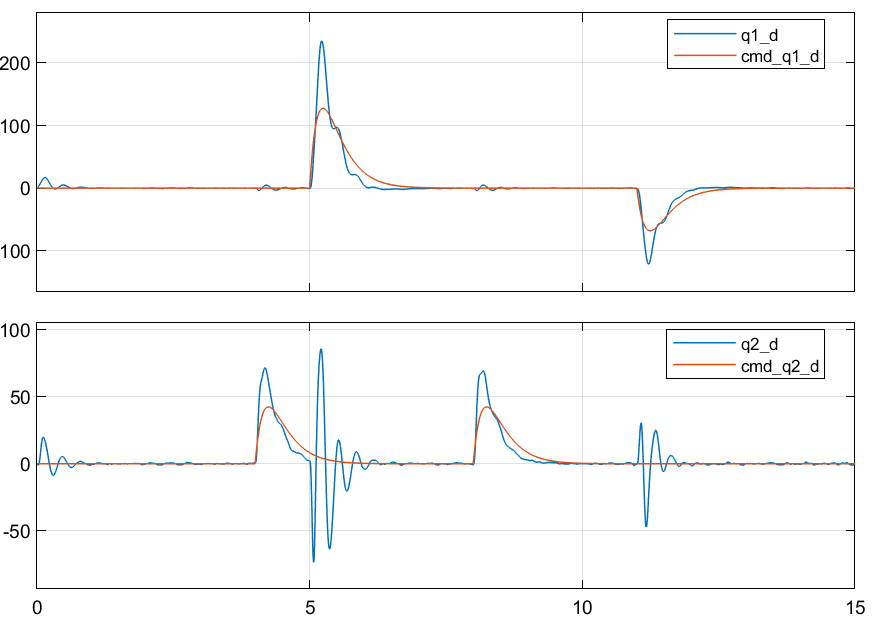
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Figure 79: Pole placement+D control realistic system step response - joint velocity – with parameter deviation

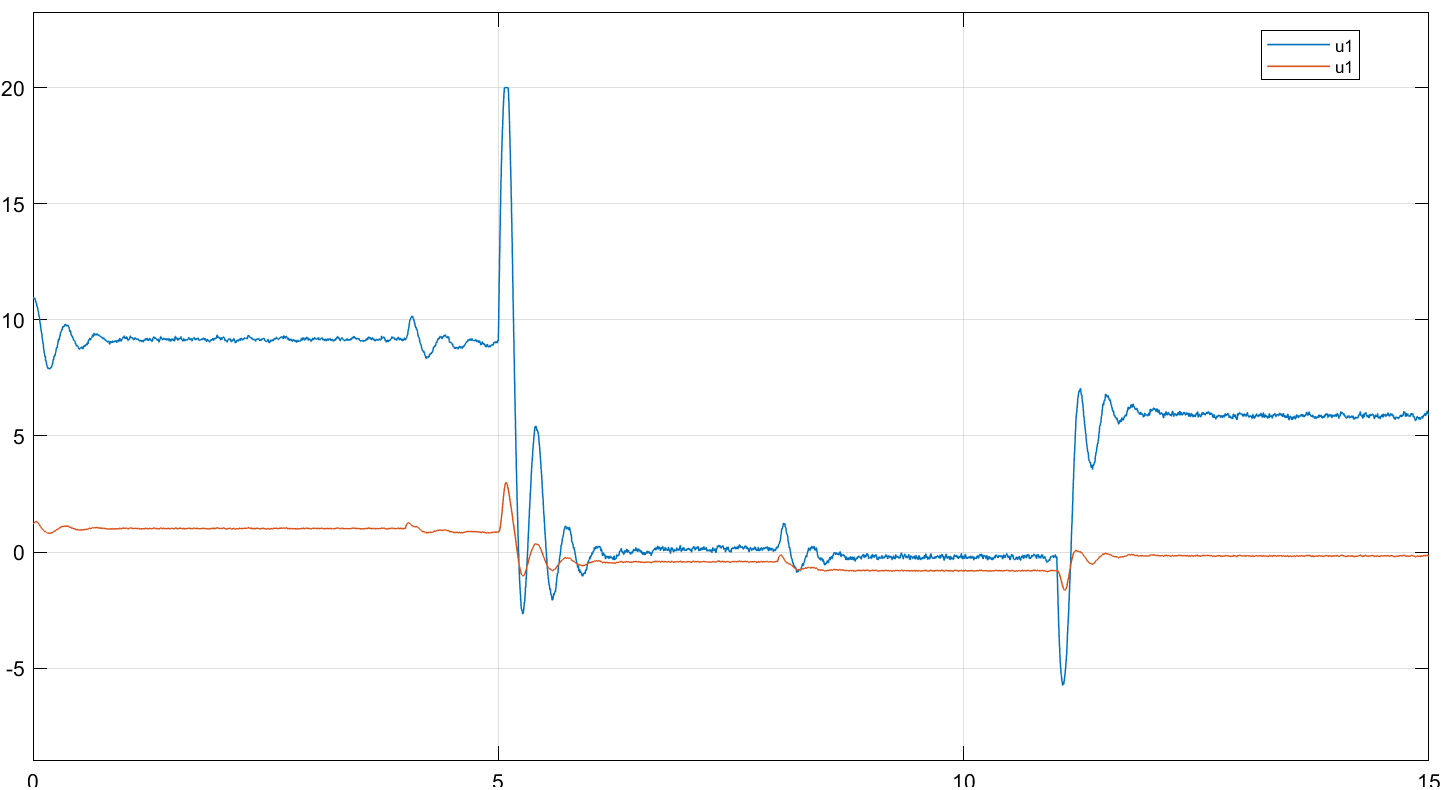


Figure 80: Pole placement+D control realistic system step response - joint input torque – with parameter deviation

### Sine wave response

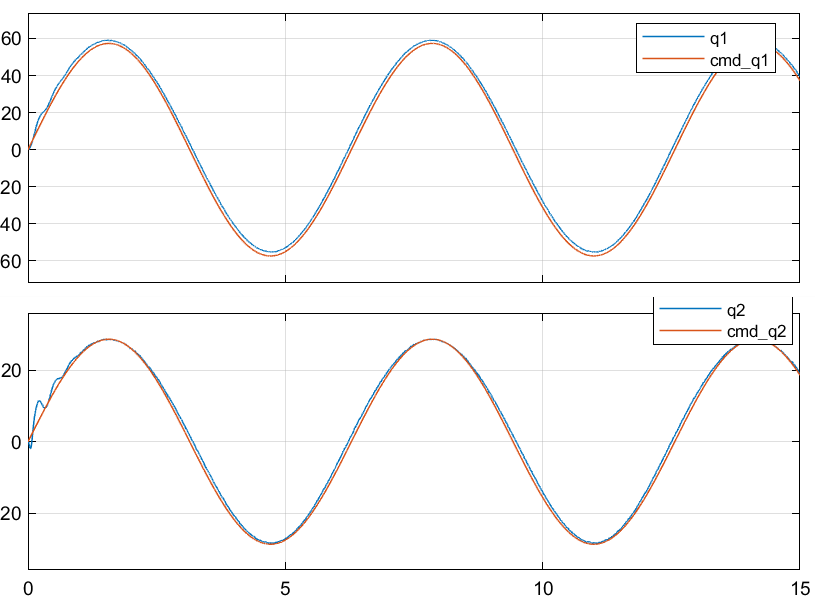


Figure 81: Pole placement+D control realistic system sine wave response - joint position – with parameter deviation

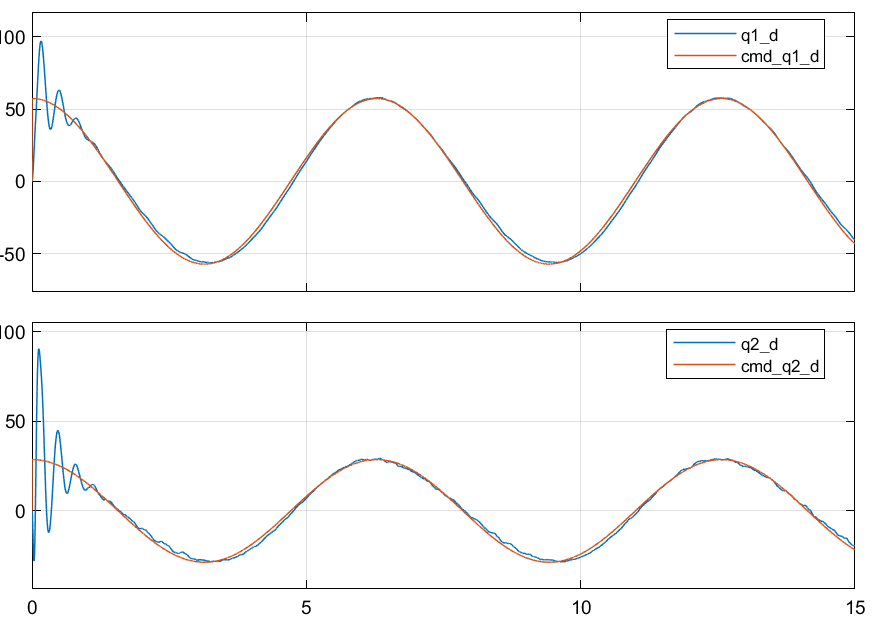


Figure 82: Pole placement+D control realistic system sine wave response - joint velocity – with parameter deviation

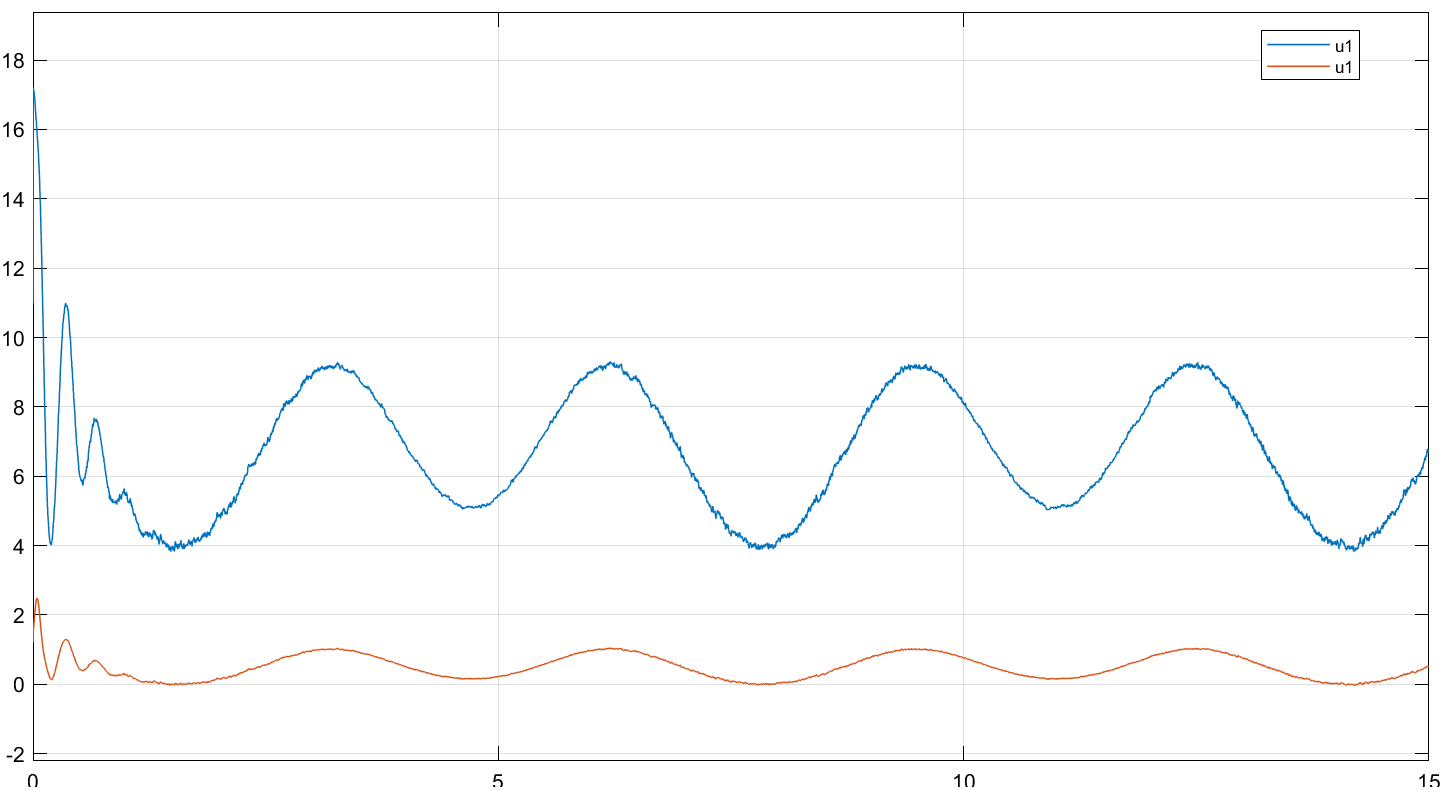


Figure 83: Pole placement+D control realistic system sine wave response - joint input torque – with parameter deviation

### Observations

* Convergence to steady-state takes longer than when there is no parameter deviation
* Overshoots and oscillations are bigger than when there is no parameter deviation
* Tracking error is worse but still acceptable

## PD+Gravity compensation Joint control

### Step response

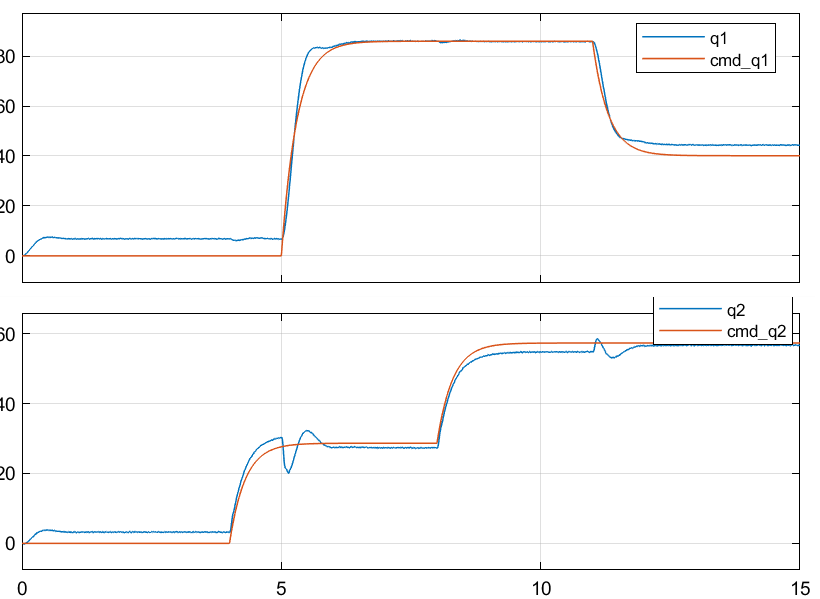


Figure 84: GRAVITY+PD control realistic system step response - joint position – with parameter deviation

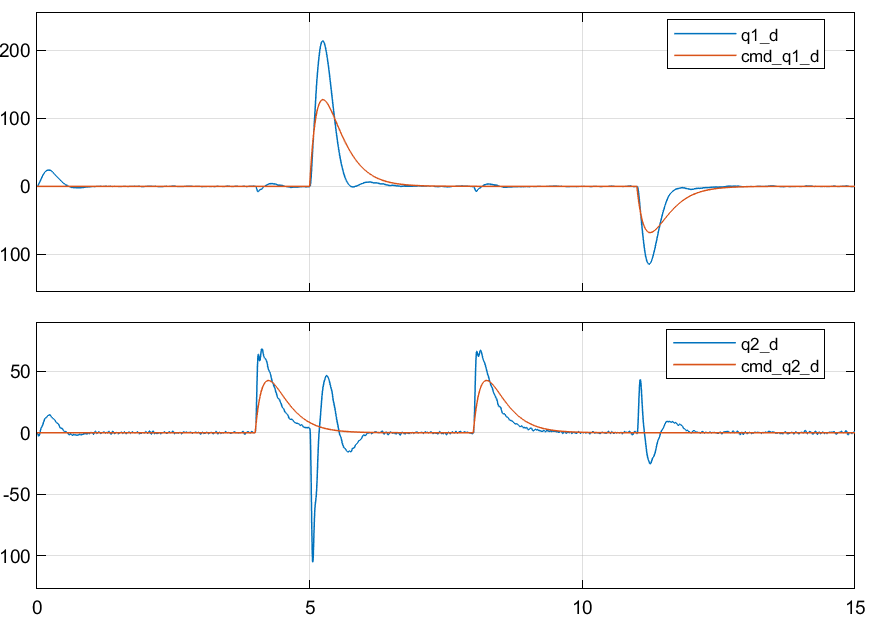


Figure 85: GRAVITY+PD control realistic system step response - joint velocity – with parameter deviation

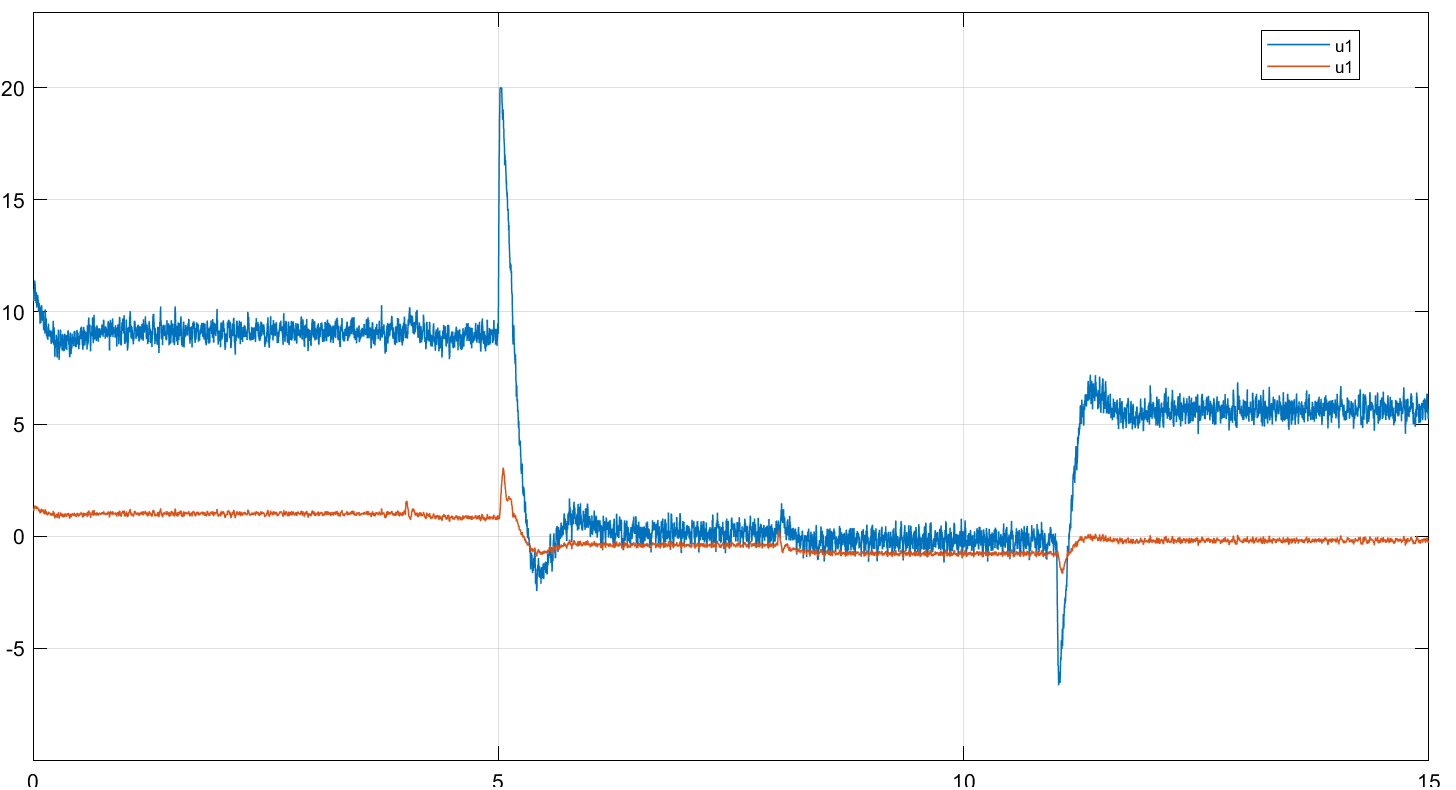


Figure 86: GRAVITY+PD control realistic system step response - joint input torque – with parameter deviation

### Sine wave response

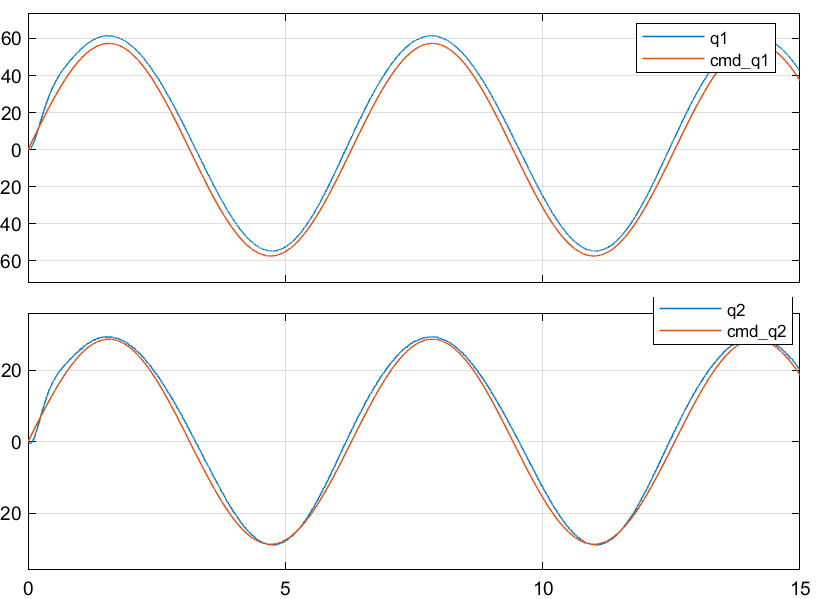


Figure 87: GRAVITY+PD control realistic system sine wave response - joint position – with parameter deviation

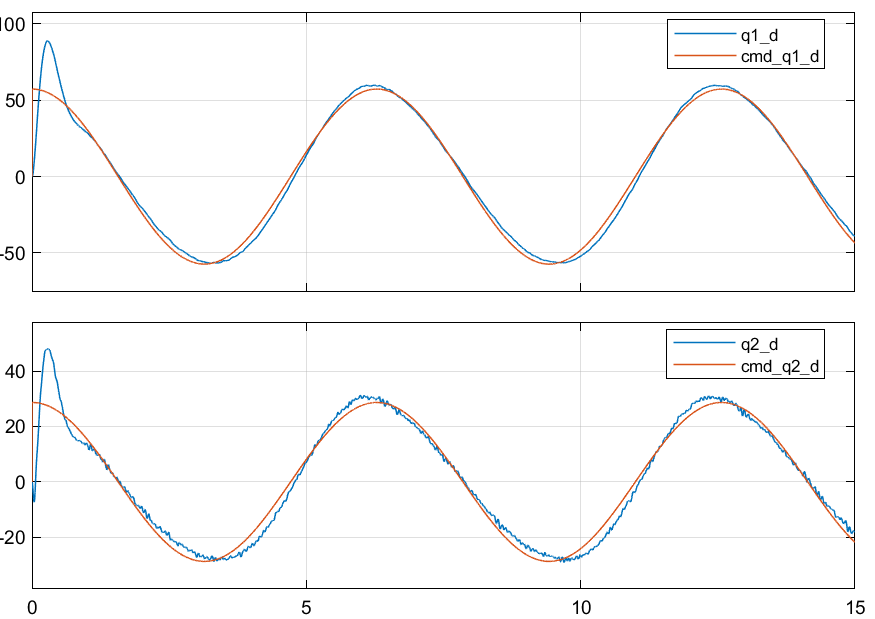


Figure 88: GRAVITY+PD control realistic system sine wave response - joint velocity – with parameter deviation

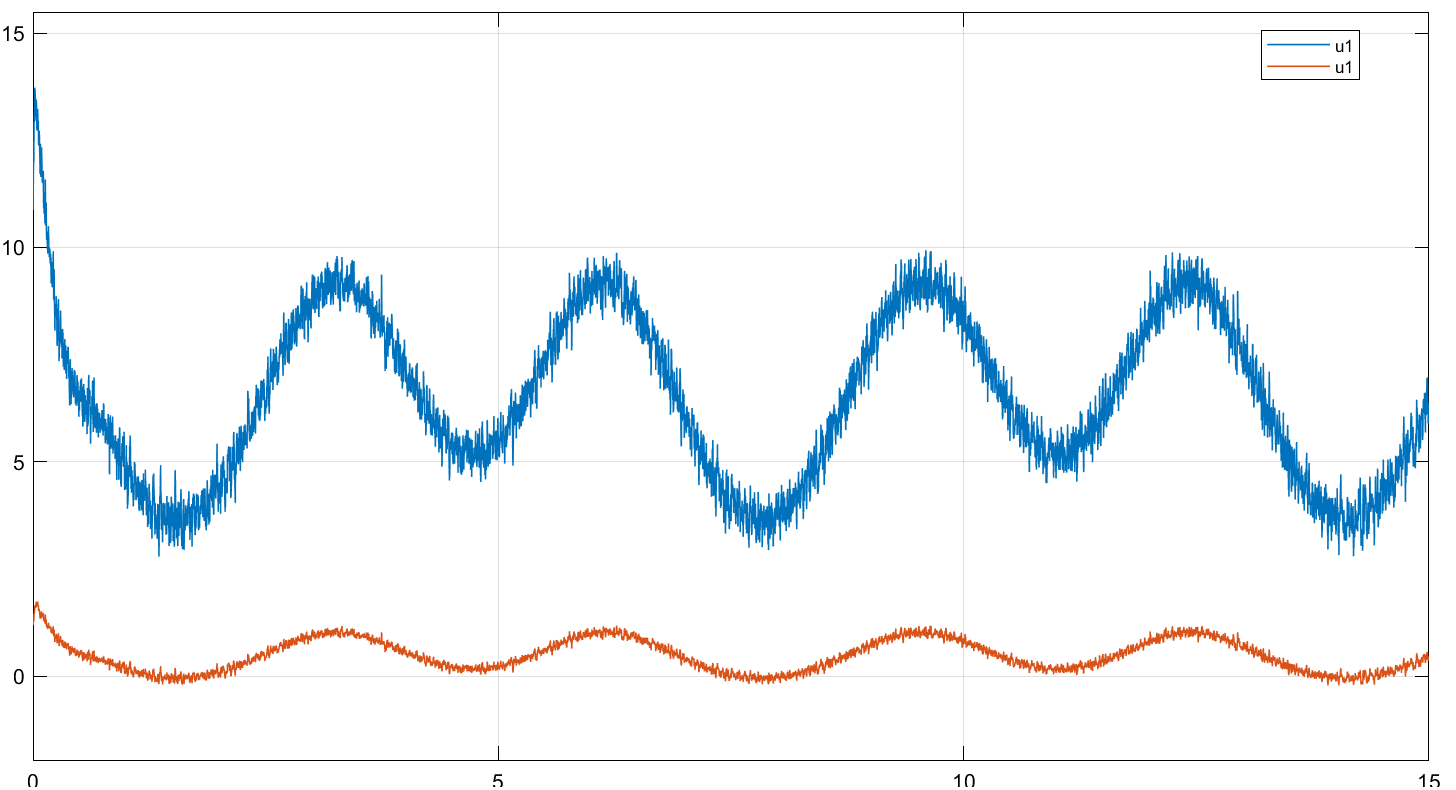


Figure 89: GRAVITY+PD control realistic system sine wave response - joint input torque – with parameter deviation

### Observations

* Bigger Steady-state error and tracking error is worse, within 4 degrees

## Inverse dynamics control

### Step response

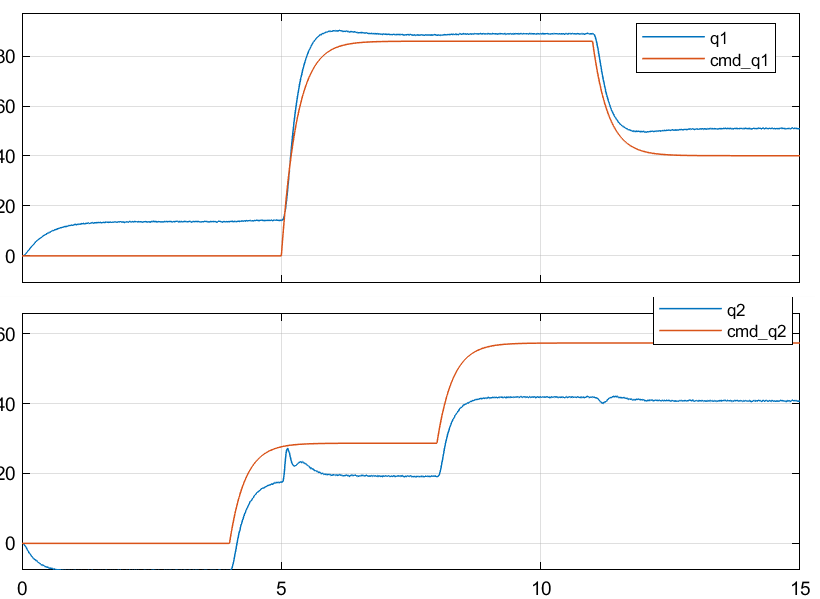


Figure 90: Inverse dynamics control realistic system step response - joint position – with parameter deviation

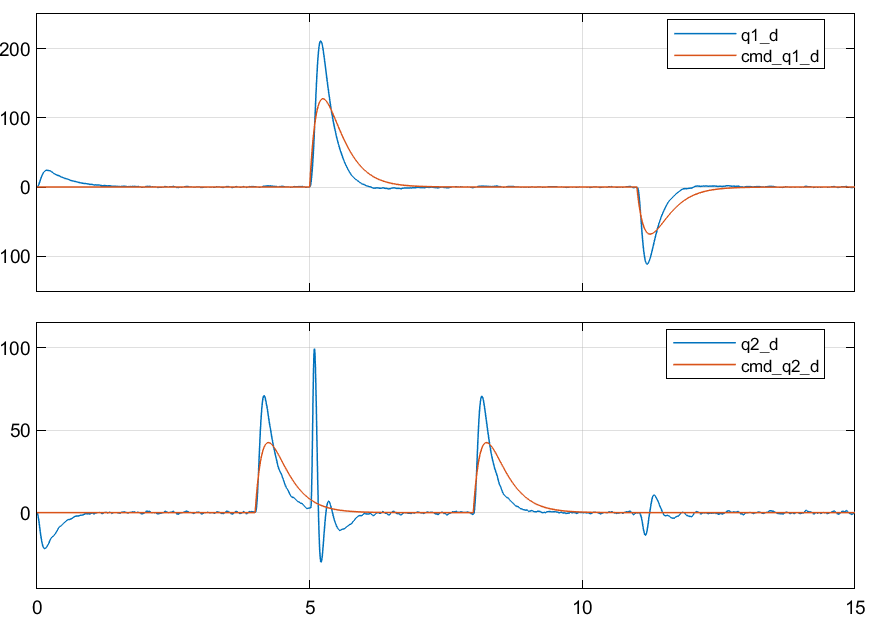


Figure 91: Inverse dynamics control realistic system step response - joint velocity – with parameter deviation

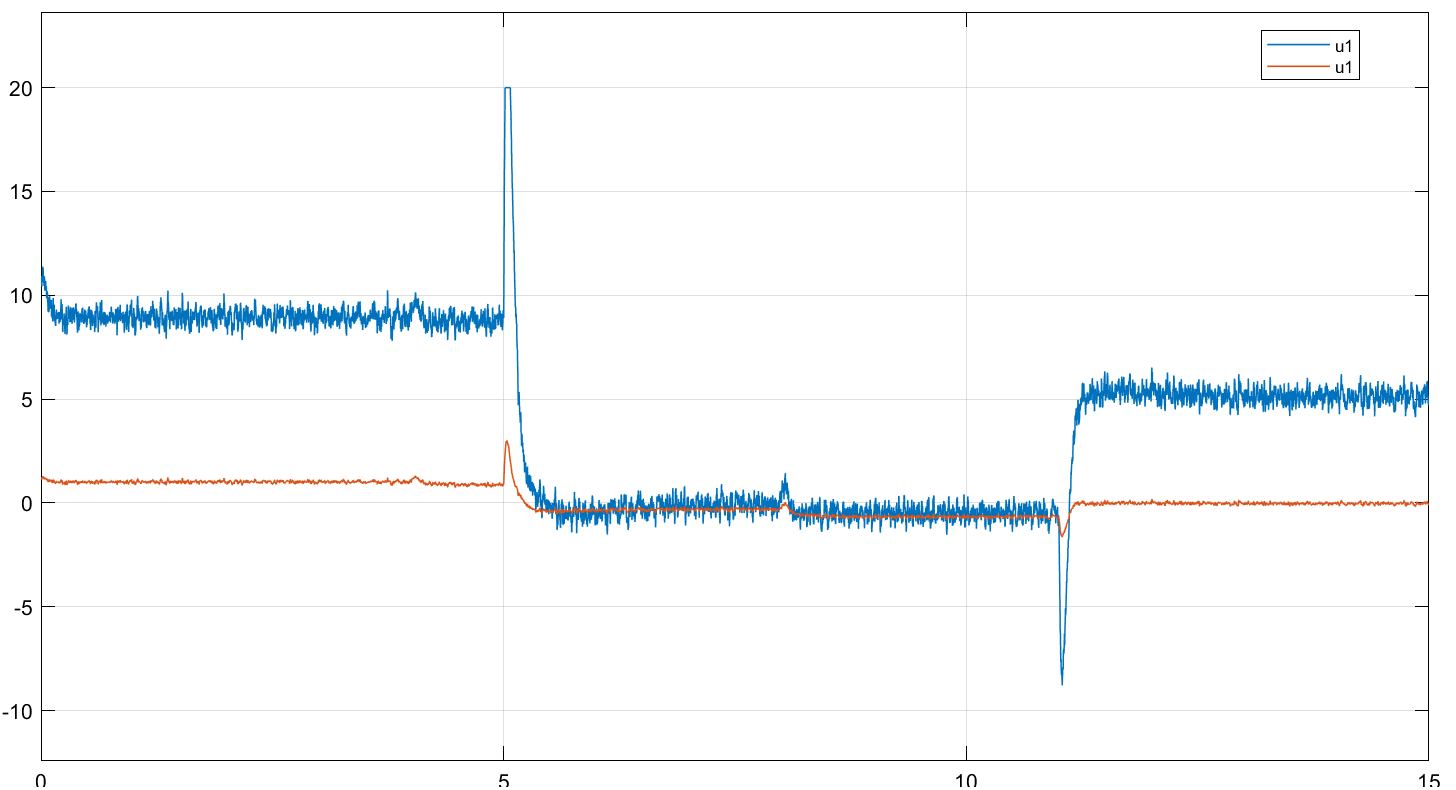


Figure 92: Inverse dynamics control realistic system step response - joint input torque – with parameter deviation

### Sine wave response

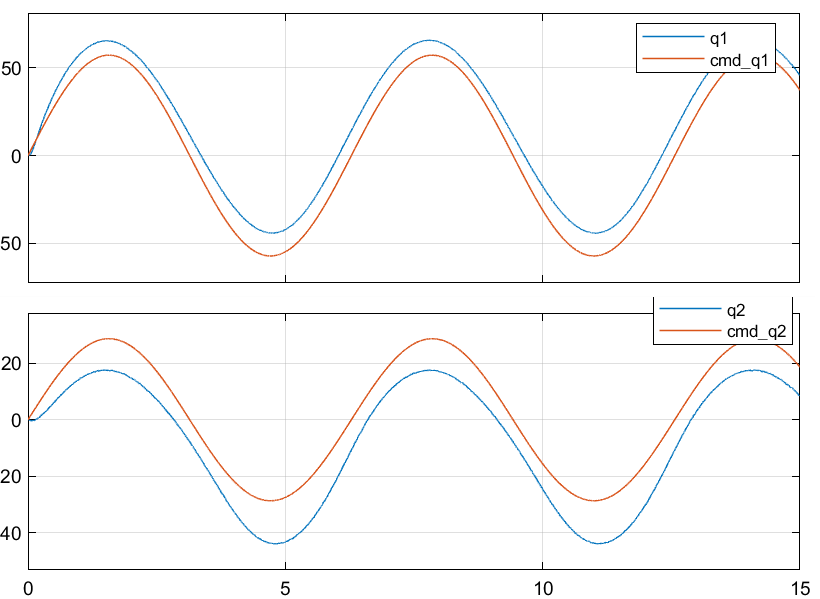


Figure 93: Inverse dynamics control realistic system sine wave response - joint position – with parameter deviation

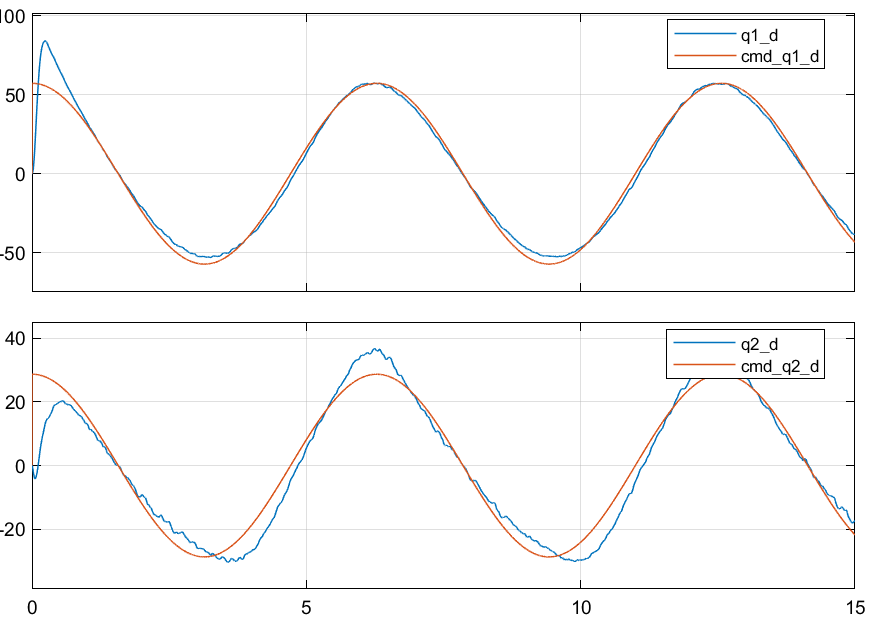


Figure 94: Inverse dynamics control realistic system sine wave response - joint velocity – with parameter deviation

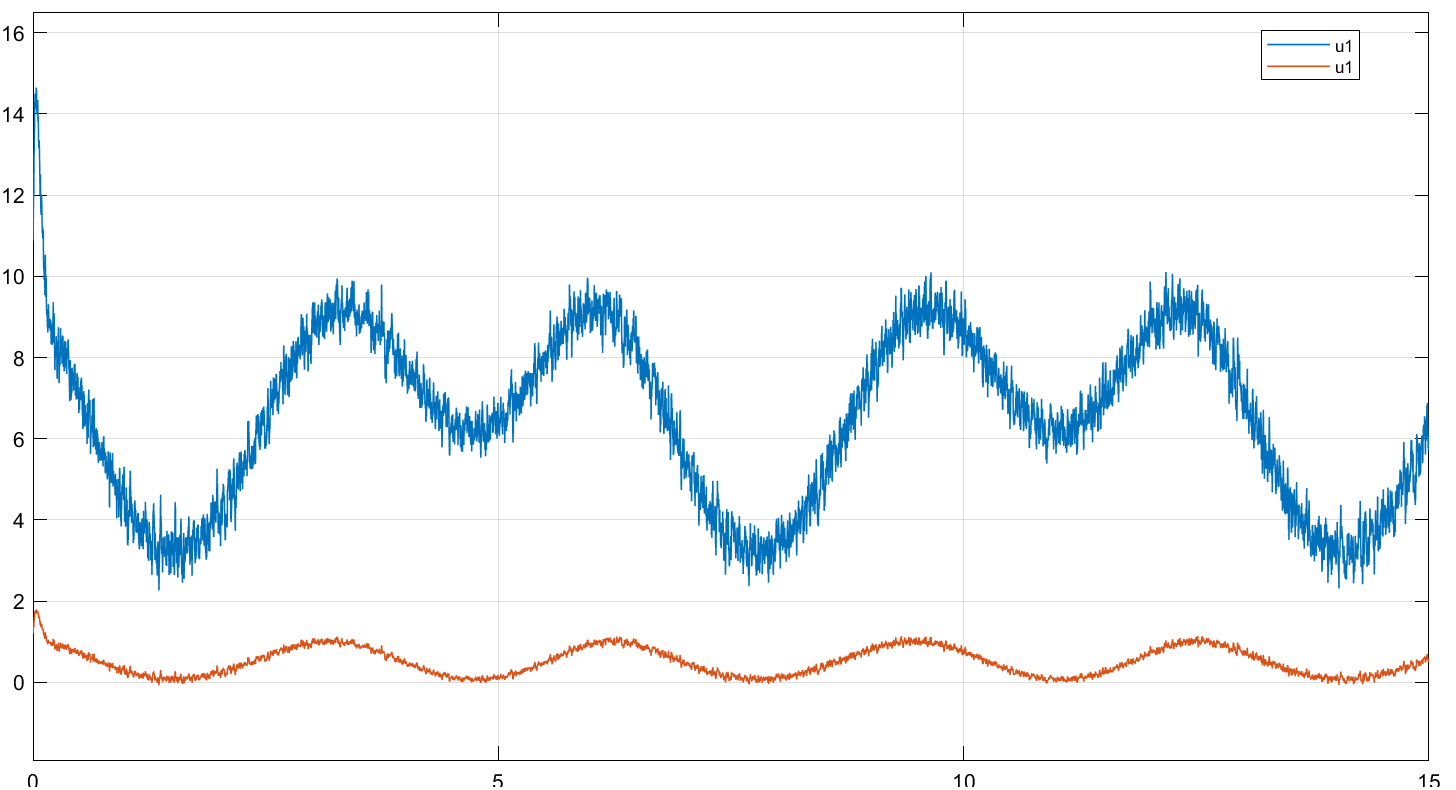


Figure 95: Inverse dynamics control realistic system sine wave response - joint input torque – with parameter deviation

### Observations

* Steady-state does not converges to zero anymore and tracking error is worse than when there is no parameter deviation

## Robust control

### Step response

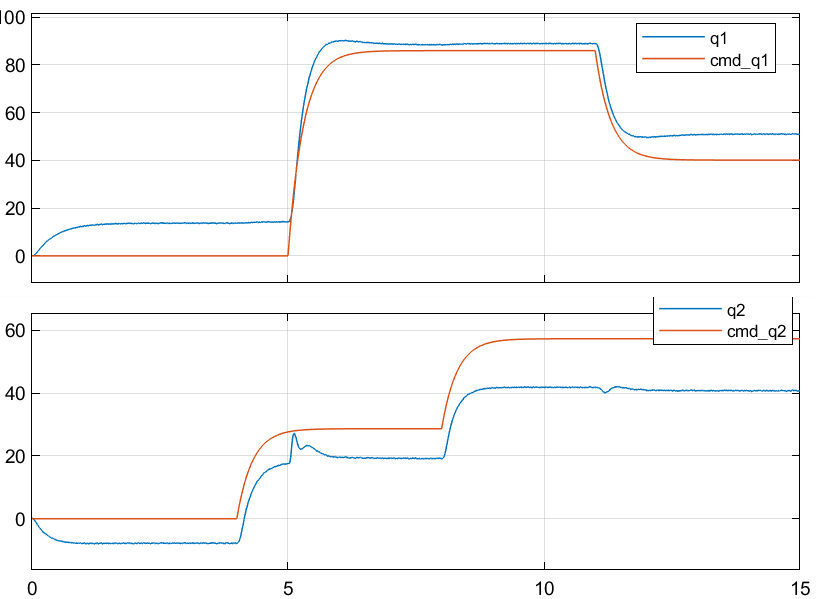


Figure 96: Robust control realistic system step response - joint position – with parameter deviation

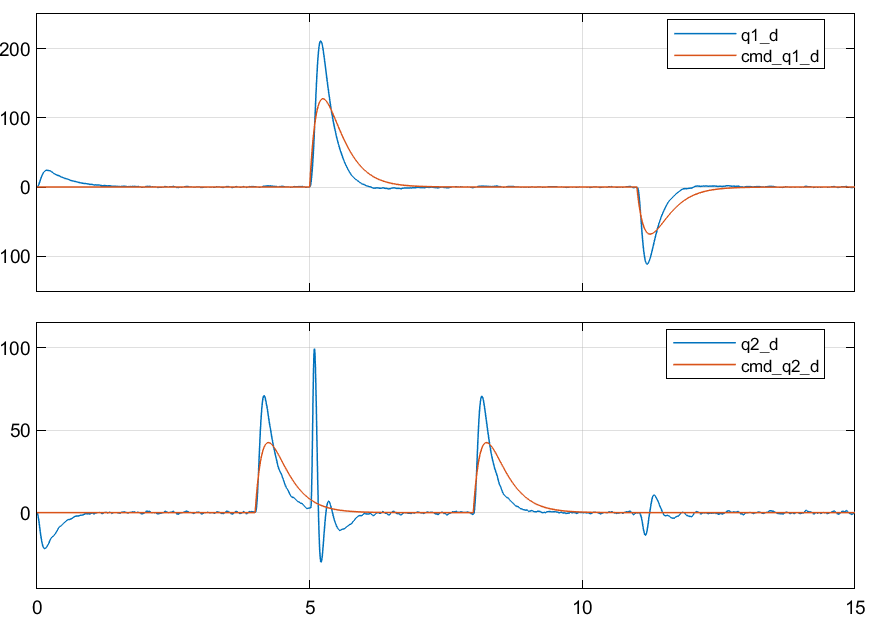


Figure 97: Robust control realistic system step response - joint velocity– with parameter deviation

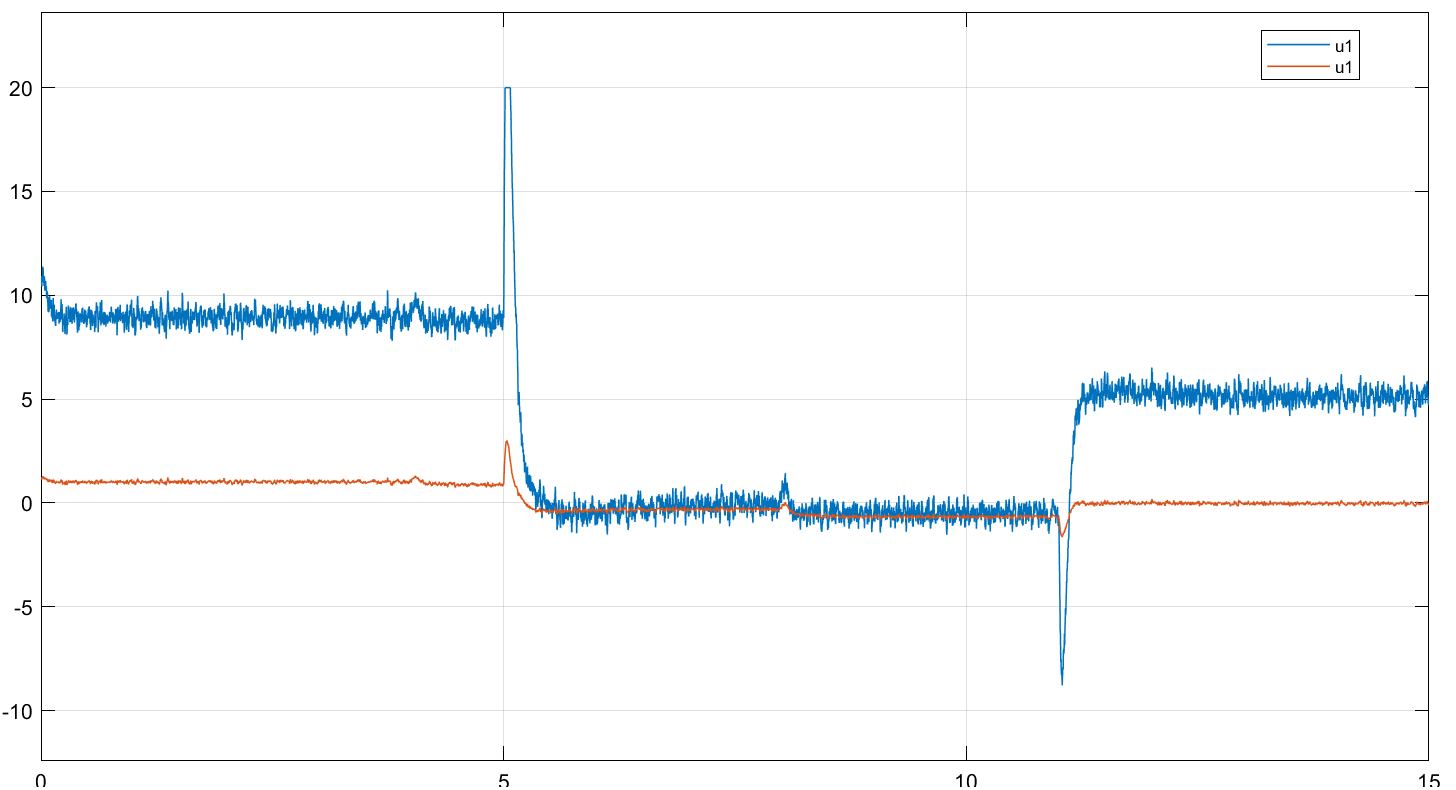
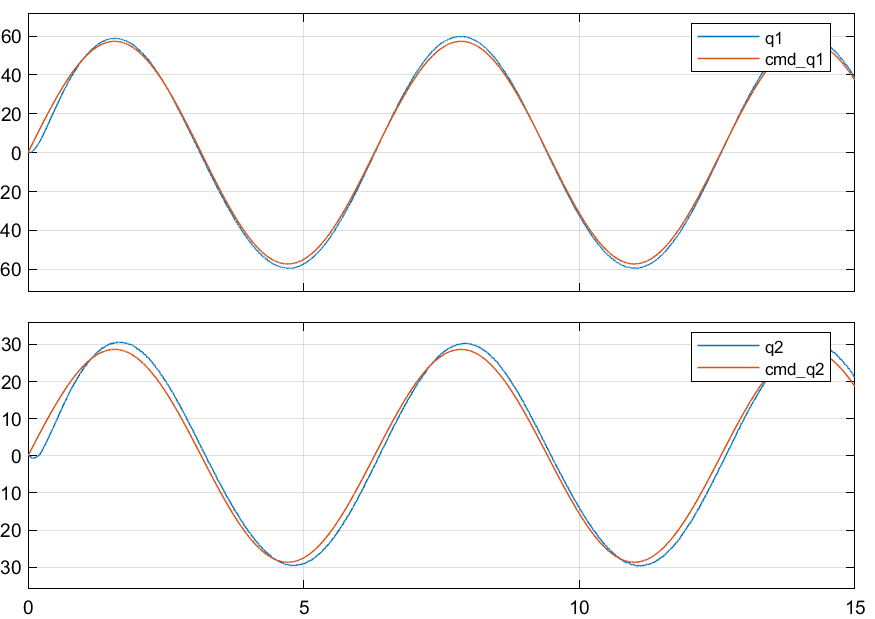


Figure 98: Robust control realistic system step response - joint input torque – with parameter deviation

### Sine wave response



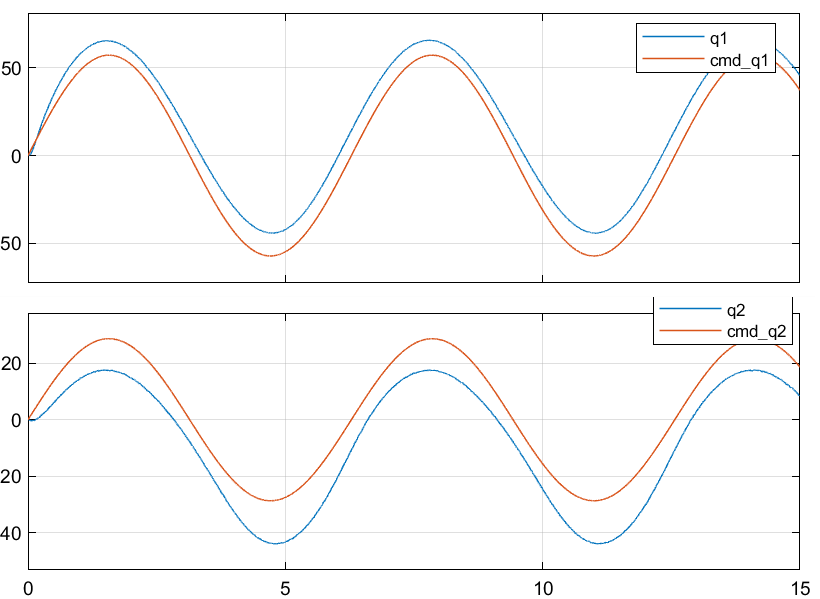


Figure 99: Robust control realistic system sine wave response - joint position – with parameter deviation

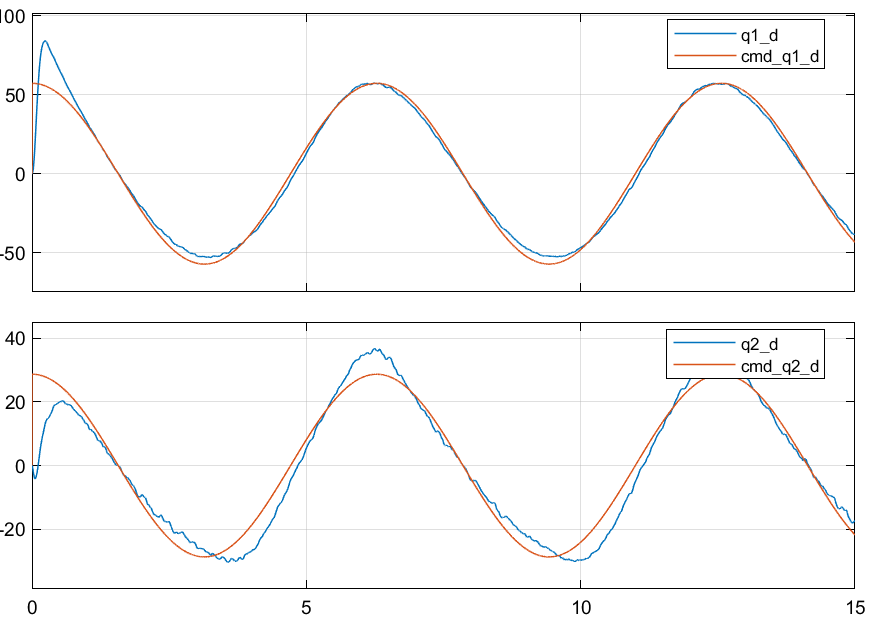


Figure 100: Robust control realistic system sine wave response - joint velocity – with parameter deviation

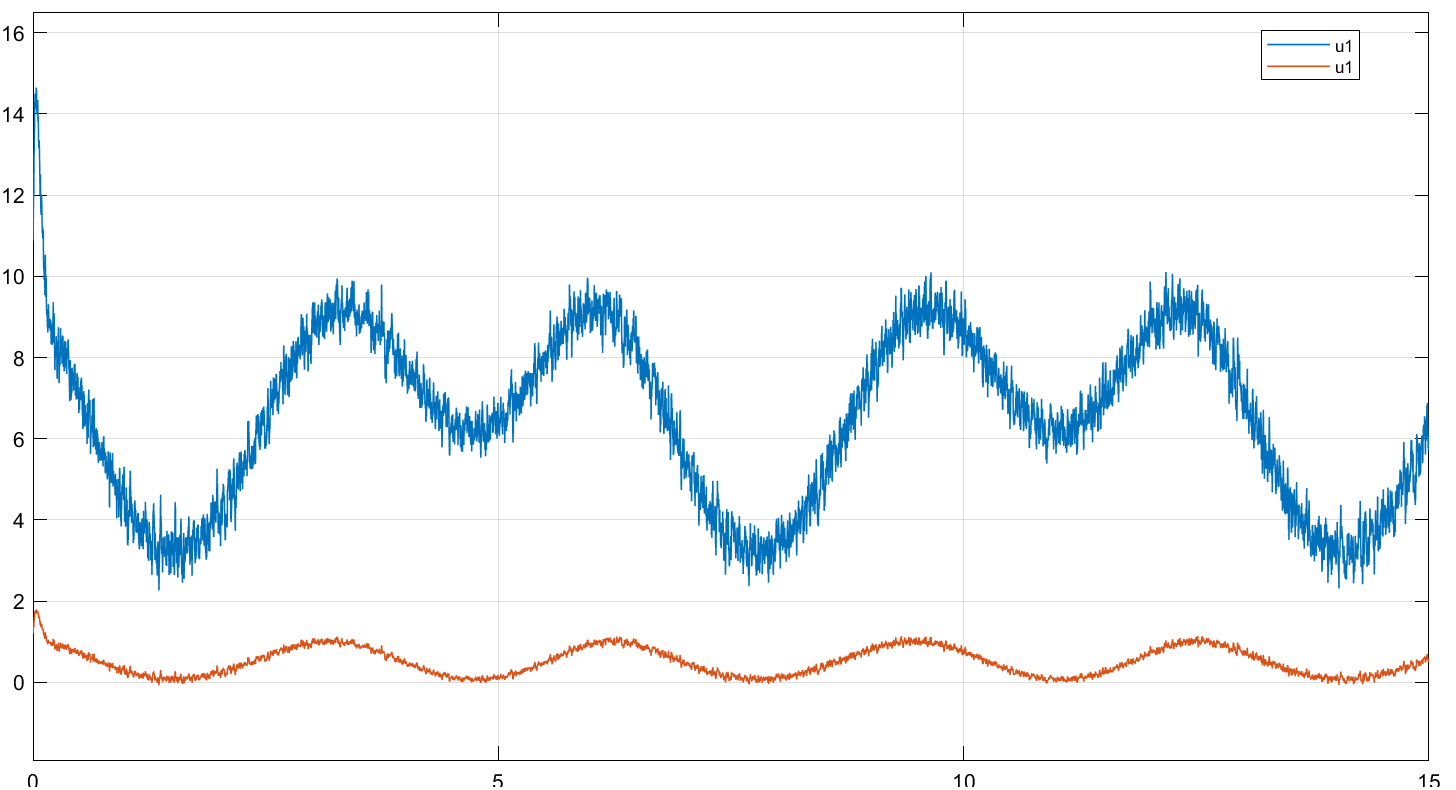


Figure 101: Robust control realistic system sine wave response - joint input torque – with parameter deviation

### Observations

* Bigger steady-state and tracking errors and it doesn’t seem to be robust against parameter deviation